

Permutations/Combinations

Formulas:

Permutations

$$P(n, r) = \frac{n!}{(n-r)!}$$

Combinations

$$C(n, r) = \frac{n!}{(n-r)!r!}$$

n = number of choices

r = number choosing

$$P(n, r): nPr$$

$$P(5, 3): 5 nPr 3$$

$$C(n, r): nCr$$

$$C(5, 3): 5 nCr 3$$

Permutations With Repetitions

For permutations involving repetitions, the number of permutations involving n objects of which p and q are alike is $n!/(p!q!)$

Example 1 How many 10-letter patterns can be formed from the letters of the word *basketball*?

The ten letters can be arranged in $P(10, 10)$, or $10!$, ways. However, some of these 3,628,800 ways have the same appearance because some of the letters appear more than once.

$\frac{10!}{2!2!2!}$ There are 2 a's, 2 b's, and 2 l's in *basketball*.

$$\begin{aligned}\frac{10!}{2!2!2!} &= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1 \cdot 2 \cdot 1} \\ &= 453,600\end{aligned}$$

There are 453,600 ten-letter patterns that can be formed from the letters of the word *basketball*.

Circular Permutations

When n objects are arranged in a circle, there are $(n-1)!$ or $n!/n$ permutations of the object around the circle.

Example 2 Six people are seated at a round table to play a game of cards.

a. Is the seating arrangement around the table a linear or circular permutation? Explain.

b. How many possible seating arrangements are there?

a. The arrangement of people is a circular permutation since the people form a circle around the table.

b. There are 6 people, so the number of arrangements can be described by $(6 - 1)!$.

$$\begin{aligned}(6 - 1)! &= 5! \\ &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \text{ or } 120\end{aligned}$$

There are 120 possible seating arrangements.

Permutations and Combinations

n = amount choosing FROM (larger number)

r = amount TO choose (usually smaller number)

n r

$$P(3,3) =$$

$$P(15,10) =$$

$$P(12,6) =$$

$$P(10,4) =$$

$$P(8,8) =$$

n r

$$C(3,3) =$$

$$C(50,11) =$$

$$C(12,5) =$$

$$C(10,2) =$$

$$C(25,5) =$$