# Matrix Applications

**Communication Matrices:**
Using matrices to show the "communication" in a network.
e.g. ranking systems; flights

**Transformation Matrices:**
Using matrices to make transformations of geometric points.
e.g. computer animation

**Transition Matrices:**
Using matrices to show change over set intervals.
e.g. population; brand loyalty; coding; game payoffs

# Communication & Transformation

**Communication matrices:**

Uses a square matrix that has zeros along the main diagonal, and is multiplied by itself for any following steps. These can be used to create rankings by showing "indirect" victories and can also show multi-stop flights.

**Transformation matrices:**

A 2x? matrix is created from the points using rows for the x- and y-coordinates. To translate, add a translation matrix, to dilate, multiply by the scale factor, and to reflect or rotate, multiply by the appropriate 2x2 matrix:

reflect over x-axis: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

reflect over y-axis: $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

reflect over y = x line: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

rotate 90°: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

rotate -90°: $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

rotate 180°: $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

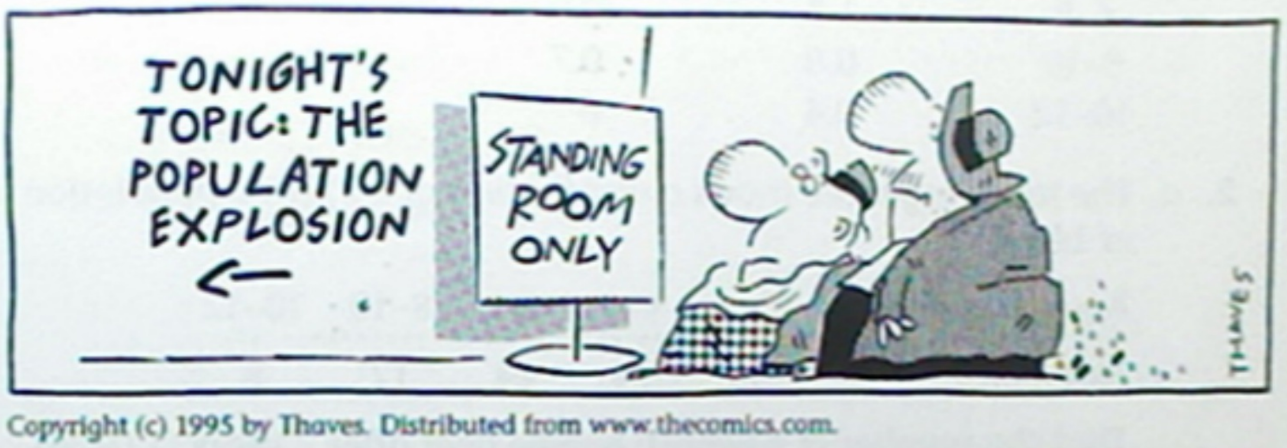
# Transformation Matrices

| 4 transformations | 4 matrix operations |
| --- | --- |
| translate (slide) → | add |
| reflect (flip) → multiply | subtract |
| rotate (turn) → | multiply |
| dilate (enlarge/reduce) → | multiply by scalar |

# Transition Matrices

**Leslie Model:**

**A square matrix that uses birth rates (first column) and survival rates (super diagonal) to determine the growth/decline of a population by multiplying the intial population by the transition matrix.**



# The *Rattus norvegicus*

**Calculating the population:**

- birth rates and survival rates are constant
- survival rate is probability to move to next age group
- life span is 18 months
- birthing begins at 3 months, then every 3 months until 15 months

| Age (months) | Birth Rate | Survival Rate |
|---|---|---|
| 0-3 | 0 | 0.6 |
| 3-6 | 0.3 | 0.9 |
| 6-9 | 0.8 | 0.9 |
| 9-12 | 0.7 | 0.8 |
| 12-15 | 0.4 | 0.6 |
| 15-18 | 0 | 0 |

| Initial population: | 0-3 | 3-6 | 6-9 | 9-12 | 12-15 | 15-18 |
|---|---|---|---|---|---|---|
| | 15 | 9 | 13 | 5 | 0 | 0 |

# Transition Matrices, cont.

**Probability matrices:**

A square matrix where the rows always equal 1 (100%). By repeated multiplying (trials), a "Markov chain" is formed that eventually results in a "steady-state matrix." The principle of experimental probability approaching theoretical probability with repeated trials is seen.

**Brand loyalty:**

Currently, 25% prefer Widgets, 60% prefer Thing-a-ma-bobs, and 15% prefer Doo-hickeys. The matrix on the right represents the percentage of change as new commercials, samples, and coupons are introduced. What are the long-term percentages?

| | W | T | D |
|---|---|---|---|
| W | .6 | .2 | .2 |
| T | .1 | .7 | .2 |
| D | .4 | .1 | .5 |

**Hospital problem:**

**In a hospital, 10% of patients are well, 70% are sick, 20% are critical, and 0% are dead. This matrix represents the percentage of patients who transition from these four phases every week. What are the long-term percentages?**

| | W | S | C | D |
|---|---|---|---|---|
| W | .9 | .1 | 0 | 0 |
| S | .6 | .3 | .1 | 0 |
| C | .2 | .4 | .3 | .1 |
| D | 0 | 0 | 0 | 1 |

# Transition Matrices, cont.

**Cryptography:**

**Using numbers to represent letters (0=space, 1=a, 2=b, etc.), a matrix is created and multiplied by an "encryption matrix" to "code" or "cipher" a message. Decoding (deciphering) is done by multiplying by the encryption matrix inverse. "Shift ciphering" can be done by beginning the code with a number of places to shift.**

**Coding example:**

**"Good morning" is coded as "7 15 15 4 0**

# Transition Matrices, cont.

**Game Theory:**

**A "payoff matrix" is formed from the perspective of the row player. The "maximin" (largest row minimum) is compared to the "minimax" (smallest column maximum). If equal, that number is the "saddle point", which is the "expected payoff" to the row player. If a saddle point exists, there is a "strictly determined" strategy.**