# Introduction to Transformations

Transformations	4 types of changes in location, direction, and/or shape
Preimage Original coordinates/shape	**Image** New/changed coordinates/shape
Translation (RIGID) "<u>SLIDE</u>" (move) in 1 or 2 directions Left→ subract from x- Right→ add to x- Up→ add to y- Dow→ subtract from y-	**Rotation (RIGID)** "<u>TURN</u>" about a fixed point Clockwise (-90 deg.) → swap x- and y- & change sign of new y- Counter-clockwise (90 deg) → swap x- and y- & change sign of new x- Half-turn (180 deg) → chage both signs
Reflection (RIGID) "<u>FLIP</u>" over line Reflect over y- → change sign of x- Reflect over x- → change sign of y-	**Dilation (NOT RIGID)** "ENLARGE"/"REDUCE" Multiply both coodinates by scale factor

Example 1: Name the type of transformation depicted in the diagram below.
Dashed figure (preimage)→solid figure(image)

_____   _____   _____   _____
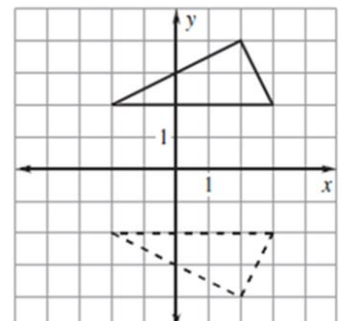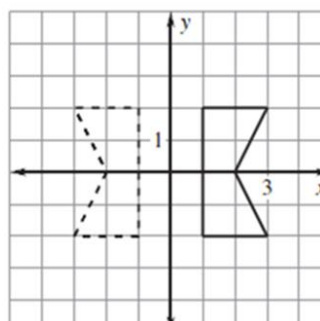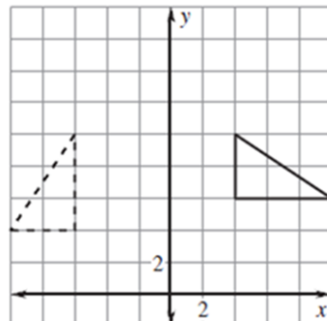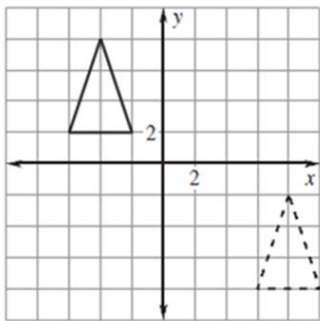
Example 2: Complete the statement using the description of the translation. In the description, points $(2,0)$ and $(3,4)$ are two vertices of a triangle.

a) If $(2,0)$ translates to $(4,1)$, then $(3,4)$ translates to _____.

b) If $(2,0)$ translates to $(-2,-1)$, then $(3,4)$ translates to _____.

Example 3: A point on an image and the transformation are given. Find the corresponding point on the original figure.

a) Point on the image: $(2,-4)$;

b) Point on the image: $(-5,-7)$;

Transformation: $(x, y) \dashrightarrow (x-4, y+3)$;

Original point: _____

Transformation: $(x, y) \dashrightarrow (x, -y)$;

Original point: _____

Translations

Translations	

Example 1: Use the translation (*x*, *y*) --> (*x* - 5, *y* + 8).

1. What is the image of $B(4, 2)$?

2. What is the image of $D(21, 5)$?

3. What is the preimage of $F'(23, \ 24)$?

4. What is the preimage of $H'(7, 25)$?

Example 2: Practice writing translation rules.

Given the preimage: A(2, 8)   B(5, -3)   C(-4, 7)

Write the image using the translation rule:

a) $(x, y) \dashrightarrow (x+2, -y)$

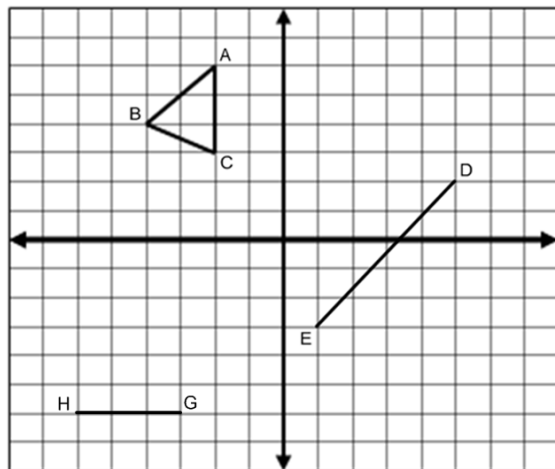**b)** $(x, y) \dashrightarrow (x-3, y+3)$

c) $(x, y) \dashrightarrow (y, x)$

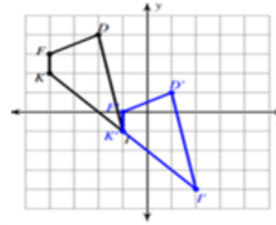**Example 3 : Translate** $\triangle ABC$ **3 units left and 2 units down.**

a. Rule? $(x, y) \dashrightarrow (x \quad , y \quad)$

b. Translate $\overline{ED}$ **: 5 units right and 6 units up.**

C. Translate $\overline{GH}$ **: 7 units left and 9 units down.**

d. Write the arrow rule for the transformation.

Example 4:

Figure *ABC* has vertices *A*(−3, 3), *B*(1, −1), and *C*(0, 5). Sketch *ABC* and draw its image after the translation (*x*, *y*) → (*x* + 4, *y* + 2).
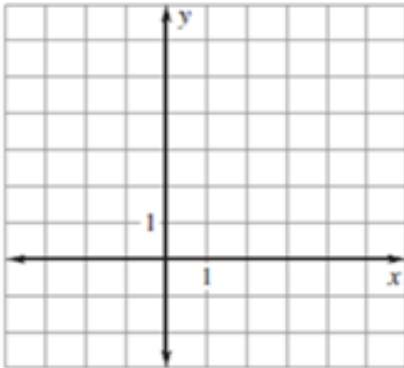
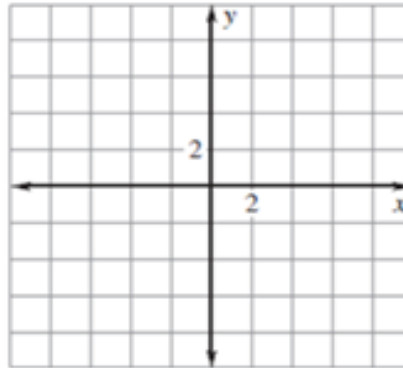Figure *ABC* has vertices *A*(4, 2), *B*(2, 6), and *C*(6, 6). Sketch *ABC* and draw its image after the translation (*x*, *y*) → (*x* − 6, *y* − 3).

Reflections

Reflection	
Axis of Symmetry	

Example 1:   Use a reflection in the *y*-axis to draw the other half of the figure.
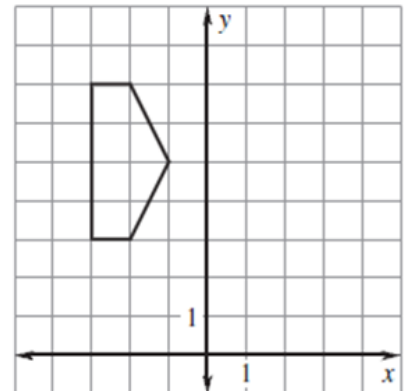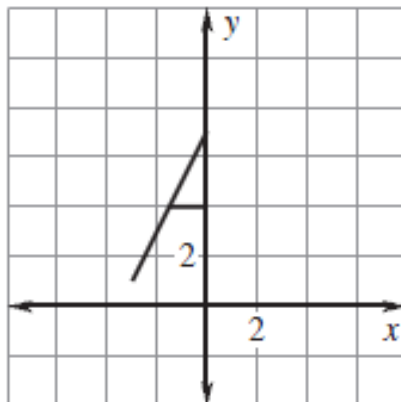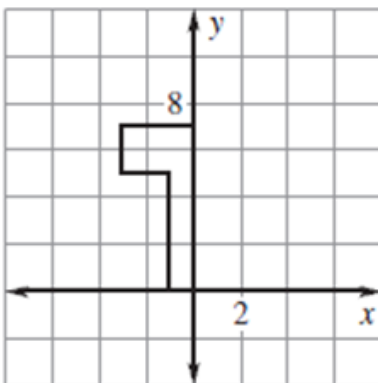
Investigation for reflection rules:

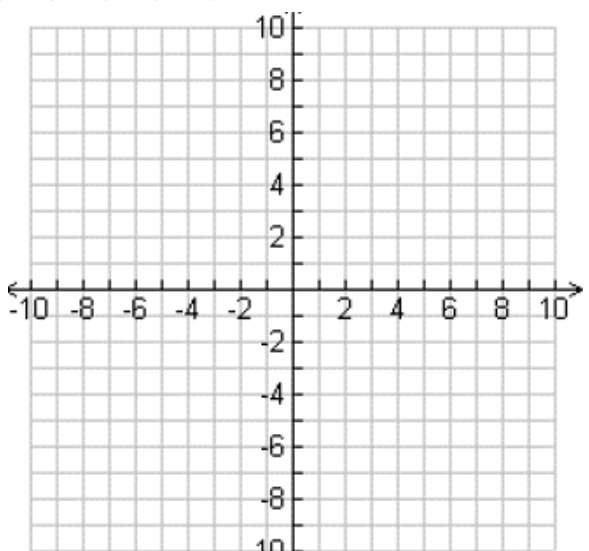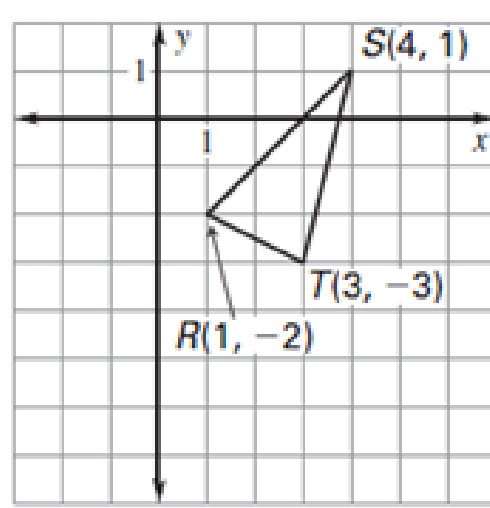| Starting Point	Reflect over x axis	Reflect over y axis	Reflect over y = x	Reflect over y = -x
A (1 , 4)				
B (5 , 2)				
C (2 , 0)				
What happened?				
Rule	$(x, y) \rightarrow ($, $)$	$(x, y) \rightarrow ($, $)$	$(x, y) \rightarrow ($, $)$	$(x, y) \rightarrow ($, $)$

Find the coordinates of the image of the figure using the given transformation.

1. Reflection across the x axis	2. Reflection across the y axis

3. Reflection across the line y = − x	4. Reflection over the line y = x
$T(2, 2), C(2, 5), Z(5, 4), F(5, 0)$	A (1,2) B (-3,4) C (-2,-8)
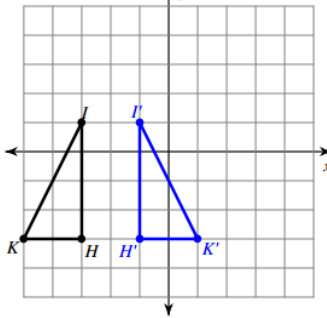
Practice:

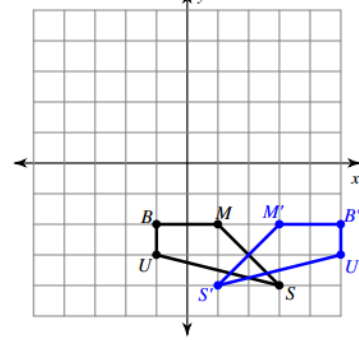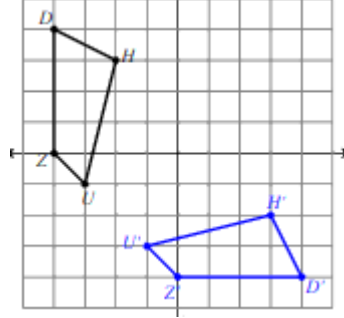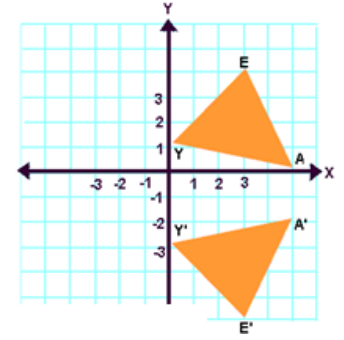| 1. Reflect the point (4,3) over the x axis.	2. Reflect the point (9,-5) over the y axis.	3. Reflect the point (0,-1) over the line y = x.	4. Reflect the point (-8,3) over the line y = −x.
5. Reflect the point (4,-2) over the line y = x.	6. Reflect the point (2, −3) over the x axis.	7. Reflect the point (-1,2) over the line y = −x	8. Reflect the point (10,3) over the y axis.

Reflections over a Line

1. Reflection across the line y = 3 D(-1,6), E(-4,6), F(-6,2)	2. Reflection across the line x = 2

Identify the Axis of Symmetry/Line of Reflection for each transformation.

Rotations

| Rotations	Turning a figure about a fixed point – the origin usually
	What ways can we "turn" objects?
	_____
	What are the two "D's" of rotation?
	_____
	Will a rotation produce a similar or congruent figure?
	_____

Example:

Rotation Rules: Use three different colored pencils and patty paper!

Starting Point	90⁰ Rotation Counter Clockwise	180⁰ Rotation Counter Clockwise	270⁰ Rotation Counter Clockwise
A (1 , 4)			
B (5 , 2)			
C (2 , 0)			
What happened?			
Rule	$(x, y) \rightarrow ($, $)$	$(x, y) \rightarrow ($, $)$	$(x, y) \rightarrow ($, $)$
What else could this be?			
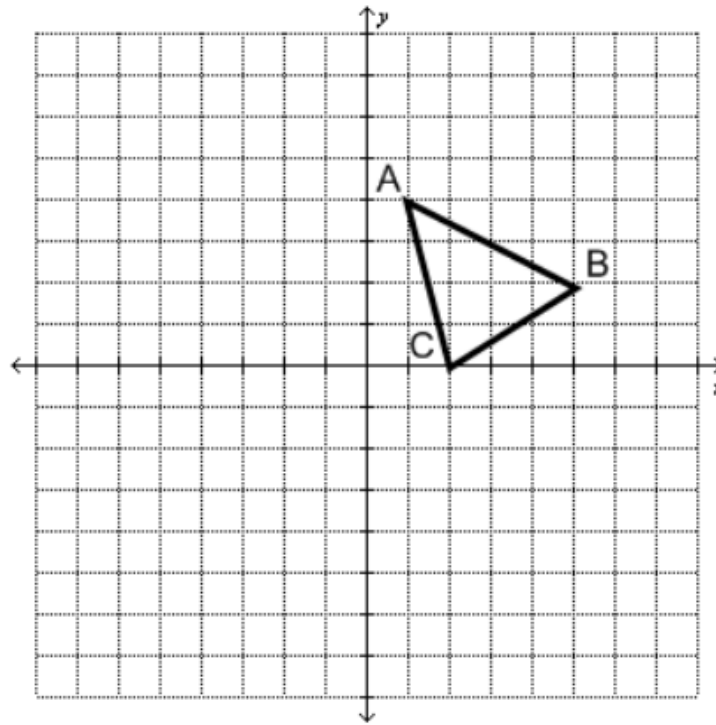
Quadrant Summary:

Example 3:

a. What are the coordinates of (3,-2) under a 90º counterclockwise rotation?	b. What are the coordinates of (-5,4) under a 180º counterclockwise rotation?	c. What are the coordinates of (3,2) under a 90º clockwise rotation?

Practice:

d. What are the coordinates of (1,3) under a 270º counterclockwise rotation?	e. What are the coordinates of (-5,6) under a 270º clockwise rotation?	f. What are the coordinates of (-7,9) under a 180º clockwise rotation?
g. What are the coordinates of (-5,3) under a 90º clockwise rotation?	h. What are the coordinates of (-8,-5) under a 180º counterclockwise rotation?	i. What are the coordinates of (7,-3) under a 90º counterclockwise rotation?

Dilations

Scale Factor	
	A transformation that produces an image that is the _____ as the original, but a _____.
	A dilation _____ or _____ the original figure.
	If the scale factor is greater than 1, the figure _____.
	If the scale factor is between 0 and 1, the figure _____.
	This transformation will NOT produce a congruent figure.

Rule: $(x, y) \rightarrow (fx, fy)$ where f represents the scale factor.

Example 1: If the scale factor is 3, how would you write the rule?

State whether a dilation using the scale factor *k* results in a *reduction* or an *enlargement*.
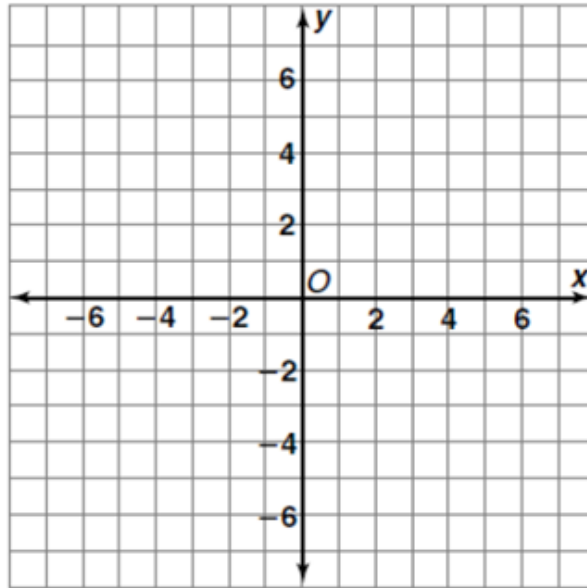
a) $k = 3$ b) $k = \dfrac{1}{3}$ c) $k = \dfrac{5}{4}$ d) $k = 0.93$
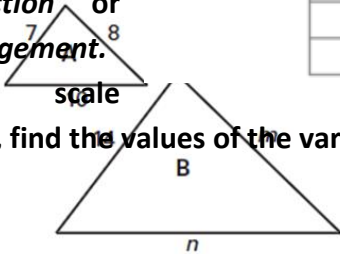
Quadrilateral *PQRS* has vertices *P* (-2, 4), *Q* (4, 4), *R* (4, -2), and *S* (- 4, - 4). It is dilated by a scale factor of ½.

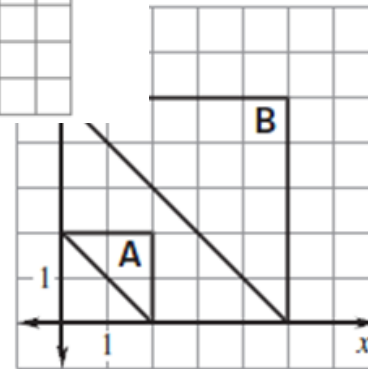**a)** What are the coordinates of the image? Graph them.

Example: dilation from A to Figure B is *reduction* or *enlargement.* the scale Then, find the values of the variables.

a)

Determine whether the Figure a an Find factor.

b)

Reduction or Enlargement?

Reduction or Enlargement?

Scale Factor?

Scale Factor?

Variables:

Example: **Write the arrow rule for the following transformations.**	
a. Translate 7 units left, 4 units down, and reflect over y axis	b. Translate 3 units right, 2 units up, and then dilate by 1/3

c. Rotate 180 degrees and then compressed horizontally by 1/2.	d. Translate 5 units up and stretch vertically by a factor of 3.
e. Reflect over line y = x, and dilate by 2.	f. Rotate 90 degrees clockwise and then reflect over the y axis

Composition

Definitions:

Composition	

1. Pre-image:	W(-3,-1), C(-4,-3), and H(-1,-3)	Arrow Rule:
Rotate the figure 270⁰		
Reflect the figure over the y-axis		
Translate the figure left 2 and up 4.		

2. Pre-image	G(2,1), H(0,3), and L(5,4)	Arrow Rule:
Translate the figure left 2 and up 1.		
Reflect the figure over y = -x		
Reflect the figure over the y-axis		

3. Pre-image:	G(1,2), S(3,0), and T(4,4)	Arrow Rule:

Rotate the figure 90°		
Dilate the figure horizontally by a scale factor of 4		
Translate the figure according to (x,y)→(x+2,y+2)		

4. Pre-image	F(-6,4), O(-1,4), and R(-2,2)	Arrow Rule:
Dilate the figure by a scale factor of ½		
Reflect the figure over the y-axis		
Rotate the figure 270° clockwise about the origin		