Math 2 Unit 3A Notes Name: _____

Factoring Review

Multiplying Binomial Factors

Example: $(x+4)(x-9)$ What process do you use?_____

1. (x – 3)(x + 4)	2. (2x + 4)(2x + 3)	3. (3x – 1)(x + 5)

I. **Greatest Common Factor – GCF (**A factor that ALL terms have in common)

To factor out a monomial, you "undistributed" or divide out the GCF: GCF (left over stuff)

Examples:

A. $3x^2+6x$	B. $4x^2+8x+4$
C. $16x^2y^2+8x^2$	D. $15x^3y^5-10x^2y^6+20x^5y^3$

Practice:

1. $z^2-z^5+9z^{21}$	2. $10x^2-5x+20x^3$	3. $5z^2-6x+9x^2$

II. Difference of Two Squares

Requirements: You must have the ***difference*** of ***two*** perfect ***squares***.

Step 1: ALWAYS check for a GCF first!!!!
Step 2: Check to see if both terms are perfect squares. (What are the perfect squares???)

Step 3: $a^2 - b^2 = (a+b)(a-b)$

* This is a pattern/formula. It does not matter which parentheses is + or –

Special Note:
$a^2 + b^2$ does NOT factor and is PRIME.

Examples:

A. $8x^2 - 18$	B. $3x^2 - 27$	C. $16x^4 - 81$

Practice:

1. $81x^2 - 4$	2. $9x^2 - 49$	3. $4g^2 - 81h^2$

III. Trinomials - Coefficient of x^2 is 1 $x^2 + bx + c$

Step 1: ALWAYS check for a GCF first!
Step 2: Make a set of double parentheses: (x)(x)
Step 3: Find what adds to be the middle number and what multiplies to be the last number!

Examples:

A. $x^2 + 7x + 12$	B. $x^2 - 5x - 36$
C. $x^2 + 10x + 16$	D. $x^2 + 2x - 24$

E. $3x^2 - 6x + 3$	F. $2x^2 - 16x + 30$

IV. Grouping - You must have 4 terms!

Step 1: ALWAYS check for a GCF first!
Step 2: Pair Up
Step 3: Take out a GCF out of BOTH pairs
Step 4: Draw two parentheses

In first parentheses, put the common factor/matching parentheses
In second parentheses, put what is left over.

Examples:

A. $x^3 - 2x^2 + 4x - 8$	B. $4x^3 - 6x^2 + 10x - 15$
C. $2ax + 2bx + 2ay + 2by$	D. $12x^3 + 2x^2 - 30x - 5$
E. $63n^3 + 54n^2 - 105n - 90$	F. $15xy + 6x^2 - 5ny - 2nx$

V. Trinomials - Coefficient of x^2 is greater than 1 $ax^2 + bx + c$

Step 1: Always check for a GCF first.

Step 2: Divide and Slide:

1. Multiply "a" down to "c"
2. Rewrite the trinomial with a = 1 and the new "c"
3. Factor using a = 1 rules (what multiplies to be the back and adds to be the middle??)
4. **Divide all numbers in parentheses by original "a"**
5. Reduce fractions and ANYTHING left in a denominator – SLIDE in front of x.

Examples:

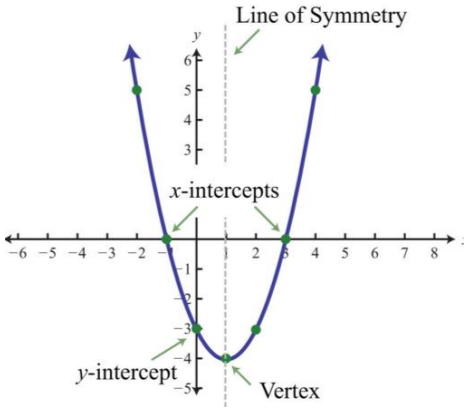
A. $6x^2+21x+9$	B. $2x^2+5x-3$
C. $6x^2-22x+12$	D. $4x^2+5x-6$

Practice:

1. $2x^2-5x-3$	**2.** $5x^2-4x-12$
3. $3x^2+16x+21$	**4.** $7x^2-9x+2$

Notes: Solving Quadratic Equations by Factoring (= 0)

I. **Quadratic Equations** The graph is a **parabola**, a u-shaped figure.



We can use the graph of a quadratic function to determine the solutions to a quadratic equation.

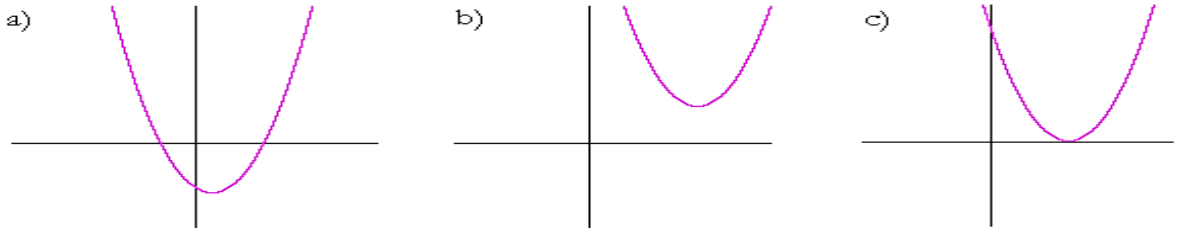
Solutions: where the graph touches/crosses the

_____.

*Terms used for solutions of quadratic equations:

_____, _____, _____.

*There are three possible outcomes when solving quadratic functions:



II. Solving a Quadratic Equation by Factoring

1. Set the equation equal to zero (move everything to one side).
2. Factor the polynomial.
3. Set each factor equal to zero and solve.

*The number of solutions _____.

Examples:

1. $(x-7)(x+3)=0$	2. $(4x+1)(3x-2)=0$
3. $x(2x-9)=0$	4. $x^2+5x+6=0$

5. $x^2 - x - 6 = 0$	6. $x^2 + 3x = 0$
7. $x^2 + 7x = -10$	8. $36x^2 = 1$
9. $5x^2 + 13x - 6 = 0$	10. $4x^2 - 25x = 21$

III. Write the quadratic Equation given the Roots.

Steps:

1. Write the factor for each solution.
2. Multiply/Simplify the factors. No fractions or decimals are allowed.

1. -5 , 6	2. $\frac{-1}{3}, 2$
3. $\frac{2}{3}, \frac{-3}{4}$	4. $\frac{-2}{5}, \frac{-1}{2}$

Notes: Solving Quadratic Equations by Square Roots and Quadratic Formula

I. Review Simplifying Radicals (Intro to Imaginary Numbers)

A. $\sqrt{72}$	B. $5\sqrt{27}$	C. $2\sqrt{242}$	D. $\sqrt{180}$
E. $4\sqrt{81}$	F. $7\sqrt{48}$	G. $\sqrt{-45}$	H. $5\sqrt{32}$
I. $\sqrt{-\frac{9}{25}}$	J. $\sqrt{\frac{-13}{16}}$	K. $\sqrt{\frac{27}{49}}$	L. $3\sqrt{\frac{-7}{81}}$

II. Solving Quadratics using the Square Root Property

Steps:

1. Isolate the variable or expression being squared. Get it by itself.
2. Square root both sides of the equation (include $\pm$ on the right).
3. Solve. You will have two solutions.

What is rational?_____ What is irrational?_____

1. $x^2 = 25$ Real (Rational or Irrational)? Imaginary?	**2.** $3x^2 = 81$ Real (Rational or Irrational)? Imaginary?	**3.** $4x^2 - 1 = 0$ Real (Rational or Irrational)? Imaginary?

4. $\frac{m^2}{15}+3=-2$ Real (Rational or Irrational)? Imaginary?	**5.** $(2y+3)^2=49$ Real (Rational or Irrational)? Imaginary?	**6.** $(3x-2)^2=48$ Real (Rational or Irrational)? Imaginary?
7. $3(x+1)^2-10=65$ Real (Rational or Irrational)? Imaginary?	**8.** $3(4x-2)^2+6=-36$ Real (Rational or Irrational)? Imaginary?	**9.** $2(2x-1)^2+4=16$ Real (Rational or Irrational)? Imaginary?

Quadratic Formula Notes:

Simplify the following rational expressions,

A. $\frac{10+\sqrt{50}}{5}$	B. $\frac{-2+\sqrt{-12}}{4}$
C. $\frac{11+\sqrt{121}}{11}$	D. $\frac{8\pm\sqrt{-36}}{2}$
E. $\frac{3\pm\sqrt{32}}{6}$	F. $\frac{3\pm2\sqrt{121}}{8}$

G. $\frac{-9 \pm \sqrt{(-5)^2 - (5)(2)(3)}}{4}$	H. $\frac{-9 \pm \sqrt{(6)^2 - 4(-3)(-3)}}{4}$

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *Use song to memorize ☺

***Put the equation in standard form first to find a, b, and c. $ax^2 + bx + c = 0$

Examples:

A. $3x^2 + 5x = 6$ A = ___ B = _____ C = _____ Real (Rational or Irrational)? Imaginary?	B. $x^2 = -6x - 9$ A = ___ B = _____ C = _____ Real (Rational or Irrational)? Imaginary?
C. $3x = -6x^2 - 4$ A = ___ B = _____ C = _____ Real (Rational or Irrational)? Imaginary?	D. $3x^2 + 5x = 2$ A = ___ B = _____ C = _____ Real (Rational or Irrational)? Imaginary?

IV. **Discriminant Formula** $b^2 - 4ac$

What is it used for?_____

Discriminant	Description of Solutions
0	
Negative	
Positive Perfect Square	
Positive Not a Perfect Square	

Find the value of the discriminant and then DESCRIBE the roots.

A. $x^2 - 10x - 50 = 0$ Discriminant:_____ Roots:_____	B. $x^2 - 21 = 4x$ Discriminant:_____ Roots:_____	C. $4x^2 - 4x + 17 = 0$ Discriminant:_____ Roots:_____

CP Notes: Complete the Square to Solve a Quadratic

Parts of Completing the Square

A. Factor each trinomial as the square of a binomial.

1. $x^2 - 2x + 1$
2. $x^2 - 10x + 25$
3. $x^2 + 12x + 36$

B. Find the value of "C" that makes the trinomial a perfect square.

1. $x^2 + 6x + "C"$
2. $x^2 - 8x + "C"$
3. $x^2 + 4x + "C"$

C. Square Root Property

Use the Square Root Property to find the X intercepts.

1. $y = (x+2)^2 - 6$
2. $y = \frac{1}{4}(x+1)^2 + 7$

D. Completing the Square - Example: $x^2 - 2x - 3 = 0$

1. Isolate the constant (move to the other side) Make two blank spaces	
2. Make sure the leading coefficient is 1 (if not, divide through!)	
3. Complete the Square (Divide the middle number by 2 and square it)	
4. Add that number to both sides of the equation.	
5. Factor the left side and combine the right side	
6. Square root both sides of the equal sign ***Remember to put a $\pm$ sign on the right!	
7. Solve. You should have two answers.	

Find the solutions (x intercepts) by completing the square. Identify whether the solutions are real or imaginary. If real, identify whether they are rational or irrational.

1. $x^2 - 4x + 2 = 0$ Real (Rational or Irrational)? Imaginary?	2. $x^2 + 6x - 16 = 0$ Real (Rational or Irrational)? Imaginary?

3. $x^2 + 2x + 5 = 0$ Real (Rational or Irrational)? Imaginary?	4. $x^2 - 2x - 8 = 0$ Real (Rational or Irrational)? Imaginary?
5. $x^2 + 8x + 1 = 0$ Real (Rational or Irrational)? Imaginary?	6. $0 = x^2 - 8x + 15$ Real (Rational or Irrational)? Imaginary?

CP Notes: Quadratic Word Problems

1. Find two numbers whose sum is 20 and whose product is 96.	2. Two numbers differ by 6 and their product is 216. Find the numbers.
3. Find the dimensions of the rectangle if the length is one more than three times the width and the area is 154 square feet.	4. Find the dimensions of the rectangle whose length is 6 more than its width if the area is 187 feet squared
5. In a right triangle the hypotenuse is 10 m long and one leg is 2 m longer that the other leg. Find the area of the triangle.	6. The square of a number decreased by 3 times the number is 28. Find all possible values for the number.

7. Helen is making an open top box by cutting a 2 inch square from each corner of a square piece of cardboard and then folding up the remaining sides. What are the dimensions of the box if the volume is 392 in^2.	8. A square is altered so that one dimension is increased by 4, while the other dimension is decreased by 2. The area of the resulting rectangle is 55. Find the area of the original square.
9. A grassy yard 25 feet by 30 feet is surrounded by a walk of uniform width. If the area of the walk is 300 ft^2, how wide is the walk?	10. If the measure of one side of a square is increased by 2 centimeters and the measure of the other side is decreased by 2 centimeters, the area of the final rectangle is 32 centimeters. Find the measure of one side of the square.
11. Find two consecutive integers such that the sum of their squares is 421.	12. Find three consecutive integers such that the product of the first integer and the third integers is 42.