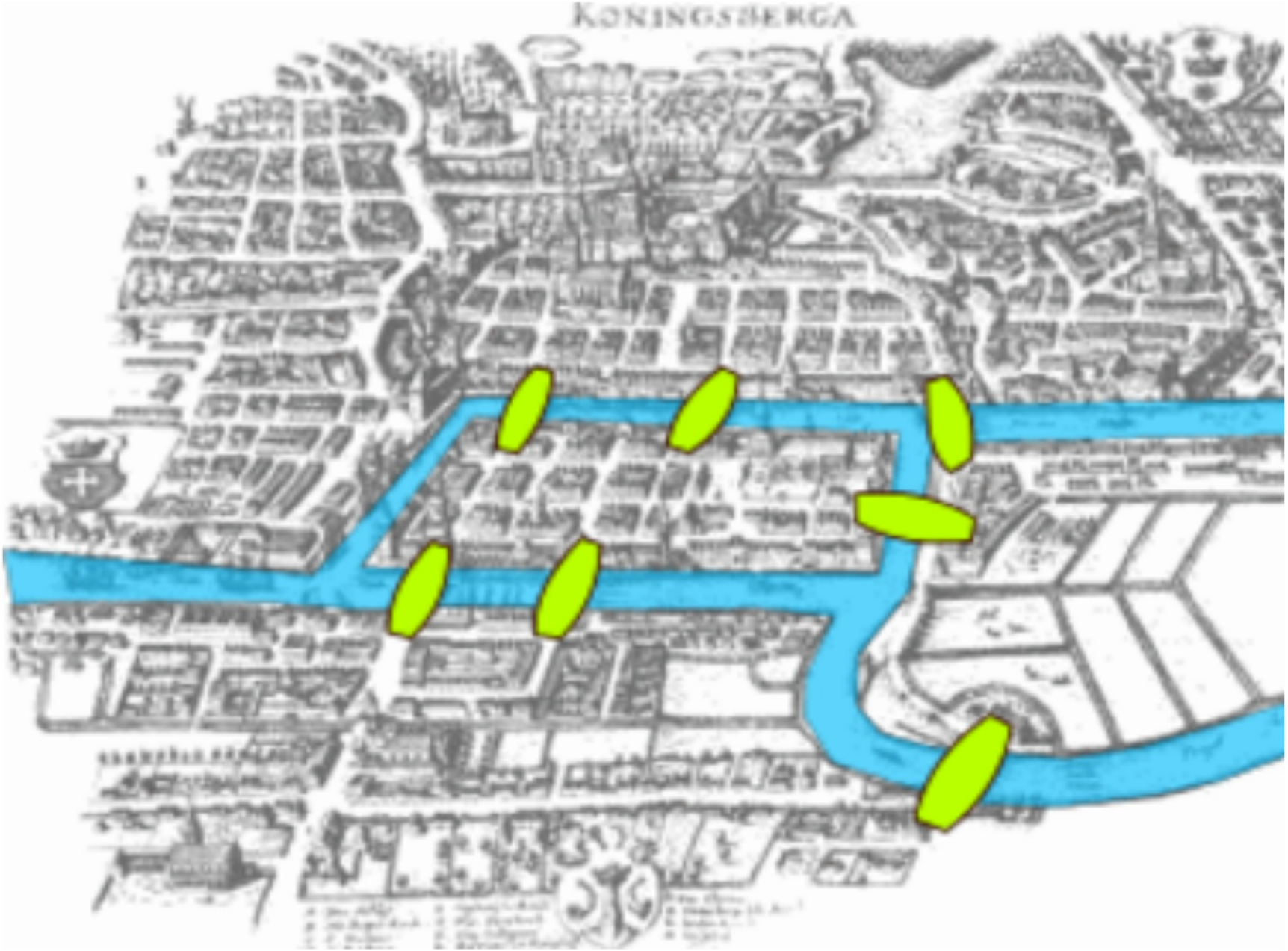
Graph Theory Topics:

- Circuits & Paths
  - Euler
  - Hamiltonian
- Digraphs (Directed graphs)
  - Tournament
- Nearest Neighbor (Greedy) Algorithm
  - Traveling Salesman problem
- Dijkstra Algorithm
- Minimal Spanning Trees (MST)
  - Prim's Algorithm
  - Kruskal's Algorithm
- 4-color Graphs
- Conflict Maps
- Critical Paths (Network Diagrams)

The 7 Bridges of Konigsberg




Kaliningrad
(Konigsberg)
Московский пр.
Московский пр.
Moskovskiy pr.
ul. Viktora Gyugo
Krasnoyarskaya ul.
Ленинский пр.
Kneiphof
Кнайпхоф
Derevyannyy most
Деревянный мост
nab. Generala Karbysheva
Königsberg Cathedral
Кафедральный собор
ulitsa Kanta
Pregolya River
Birzha
Биржа
Leninskiy pr.
Serpukhovskaya ul.
ul. Oktyabrs

Graph Theory Vocabulary:

Graph/Network/Web/Map:
diagram that models a relationship

Vertex (plural is vertices)/Node:
point with no location and no size

Edge:
connection shown with a line

Odd Degree:
vertex with an odd number of edges

Even Degree:
vertex with an even number of edges

Planar:
graph where no edges intersect

Bipartite graph:
graph whose vertices can be divided into two sets with edges having vertices in each set

Graph Theory Vocabulary, continued:

Traversable:

possibility to "walk" (move) through a graph

Path:

walk that uses each edge/vertex exactly once

Circuit:

walk that uses each edge/vertex exactly once and returns to the starting vertex

Cycle:

walk that uses each vertex and each edge exactly once and returns to the starting vertex

Euler Path/Circuit:

walk through graph using each edge only once

Hamiltonian Path/Circuit:

walk through graph using each vertex only once

Graph Theory Vocabulary, continued:

Adjacent:
connected vertices

Connected graph:
graph where all vertices are adjacent

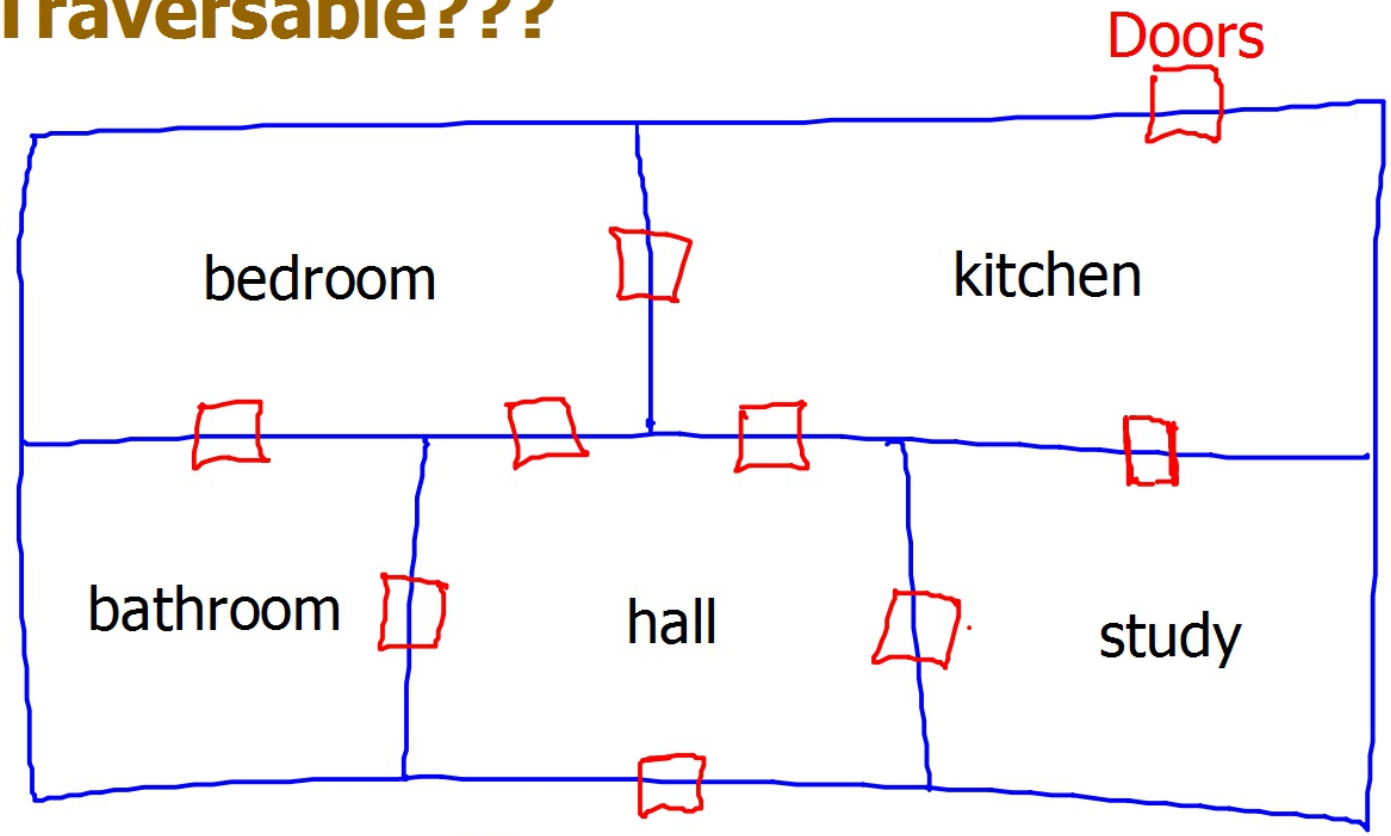
Tree:
connected graph with no cycles

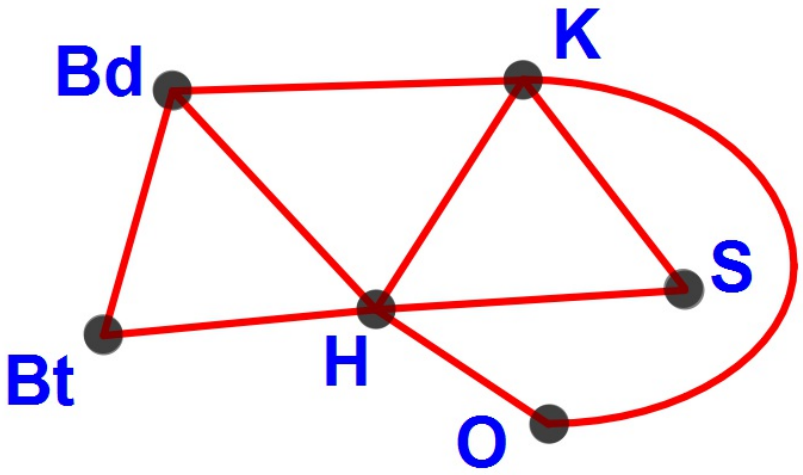
Loop:
edge that connects a vertex to itself

Relay:
edge that shows direction

Digraph:
directed graph; graph with edges showing direction of traversing

Traversable???





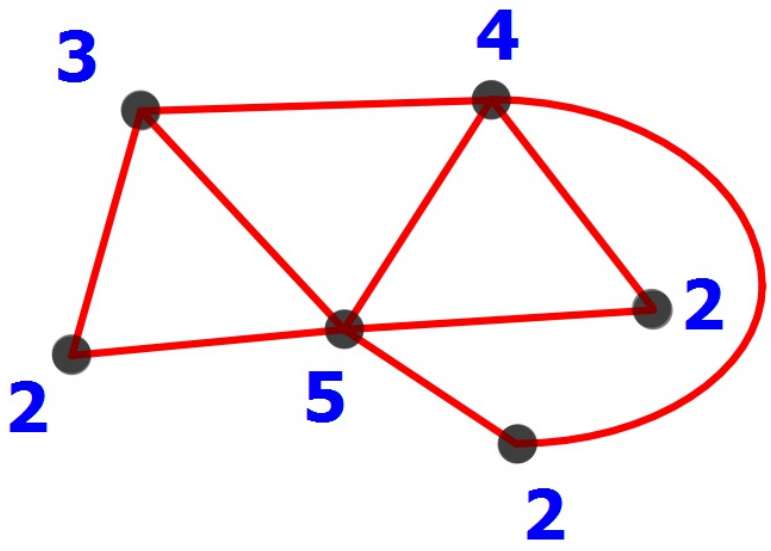
Euler Path/Circuit Rules

Path:

EXACTLY two odd vertices

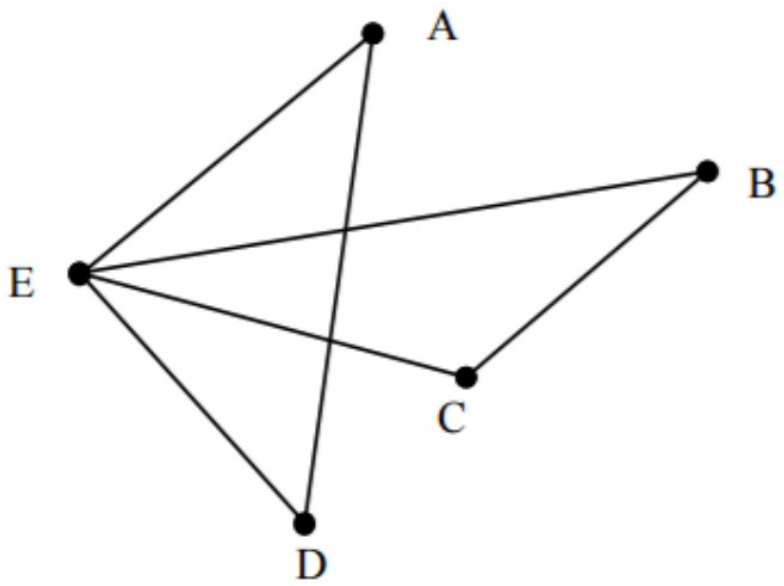
Circuit:

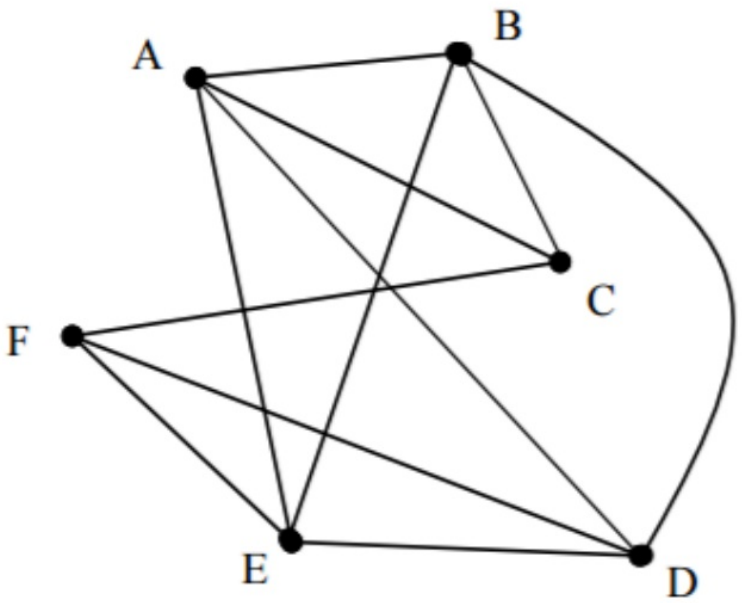
ALL vertices are even



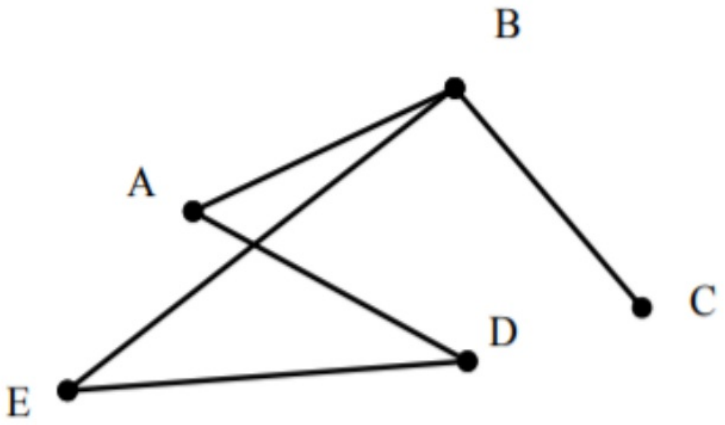
Euler/Hamiltonian Practice

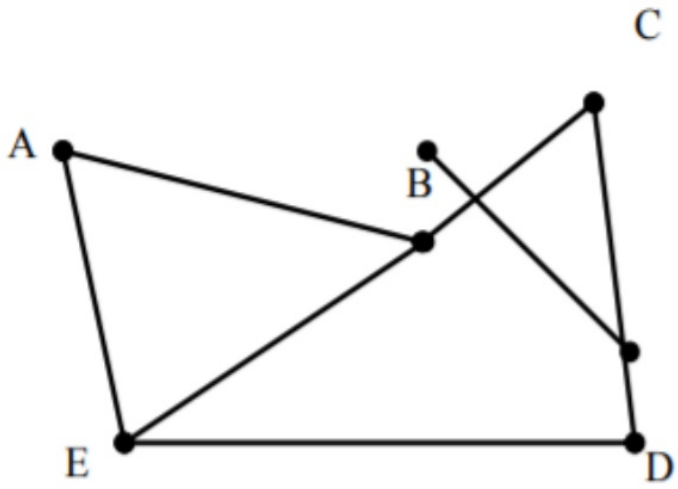
Check each graph to see if an Euler path or circuit or a Hamiltonian path or circuit can be made.





Example 9: Determine if the following graphs are bipartite. If so, list the two distinct sets of vertices.



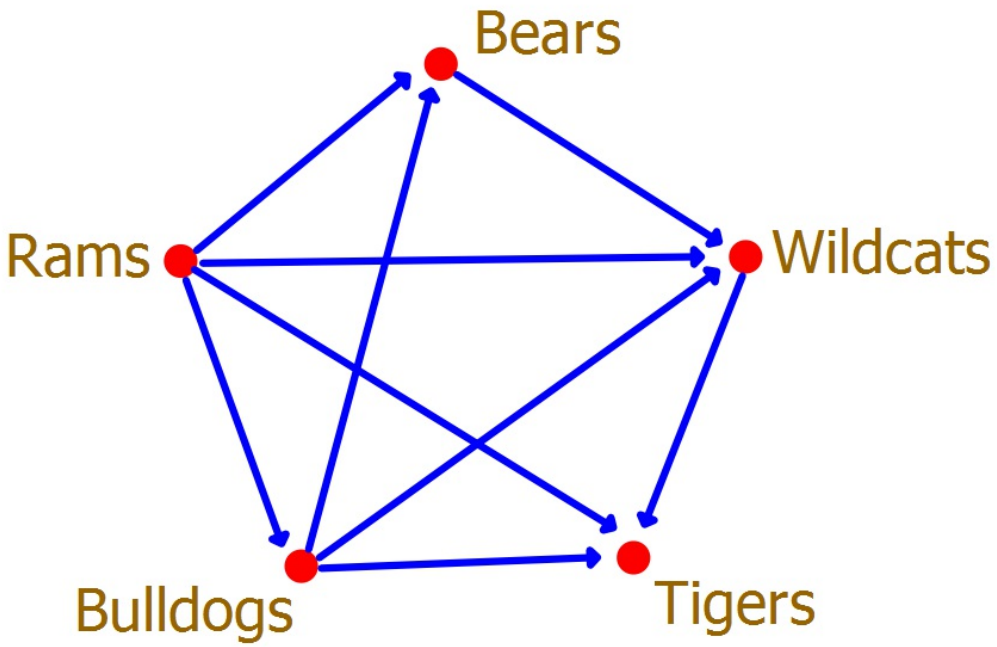


Digraphs for Tournaments

A school district has five high schools, and they each play each other in one game of football. The county needs to rank their teams.
Based on the following information about the outcomes of the games, construct a digraph and then find the ranking of the teams in the tournament.

Outcomes:

- The Bears beat the Wildcats.
- The Wildcats beat the Tigers.
- The Bulldogs beat the Bears, the Wildcats, and the Tigers.
- The Rams beat the Bears, the Wildcats, the Bulldogs, and the Tigers.

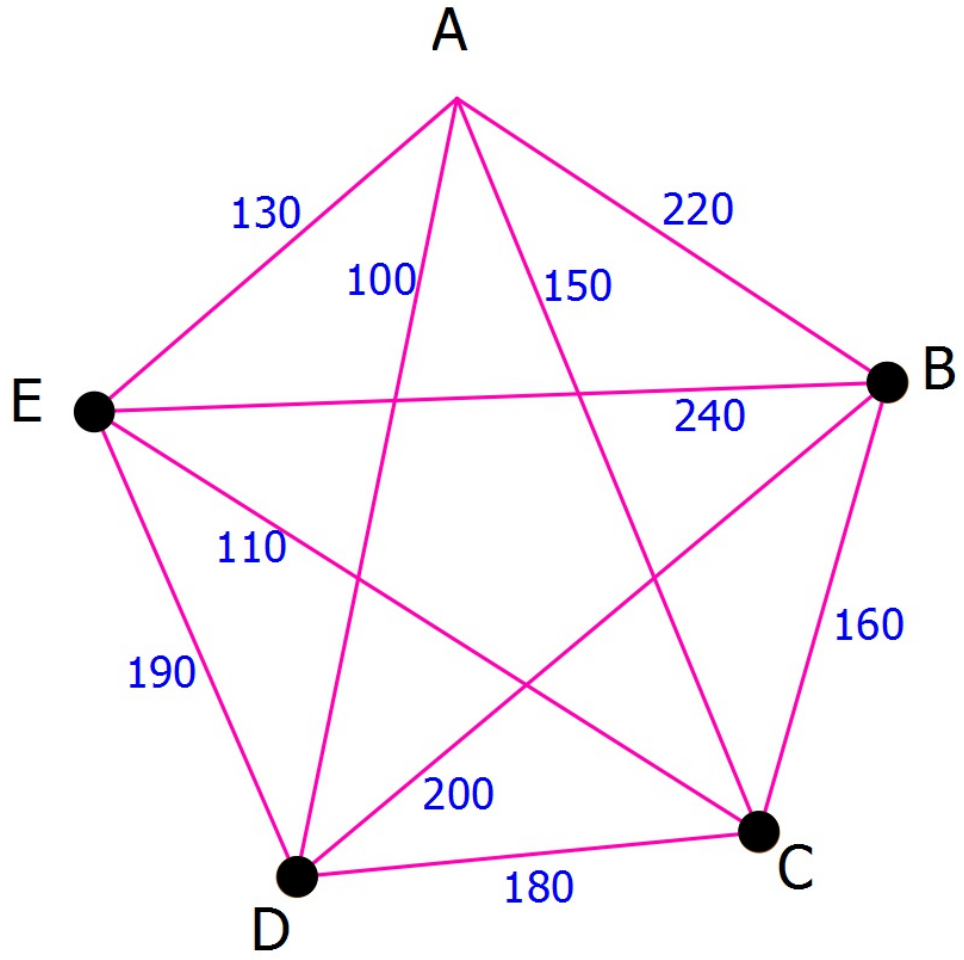


1. Rams, 4-0
2. Bulldogs, 3-1
3. Bears, 1-2
4. Wildcats, 1-3
5. Tigers, 0-3

Traveling Salesman Problem

Calculate the distance of a Hamiltonian circuit using the Nearest Neighbor (Greedy) Algorithm.

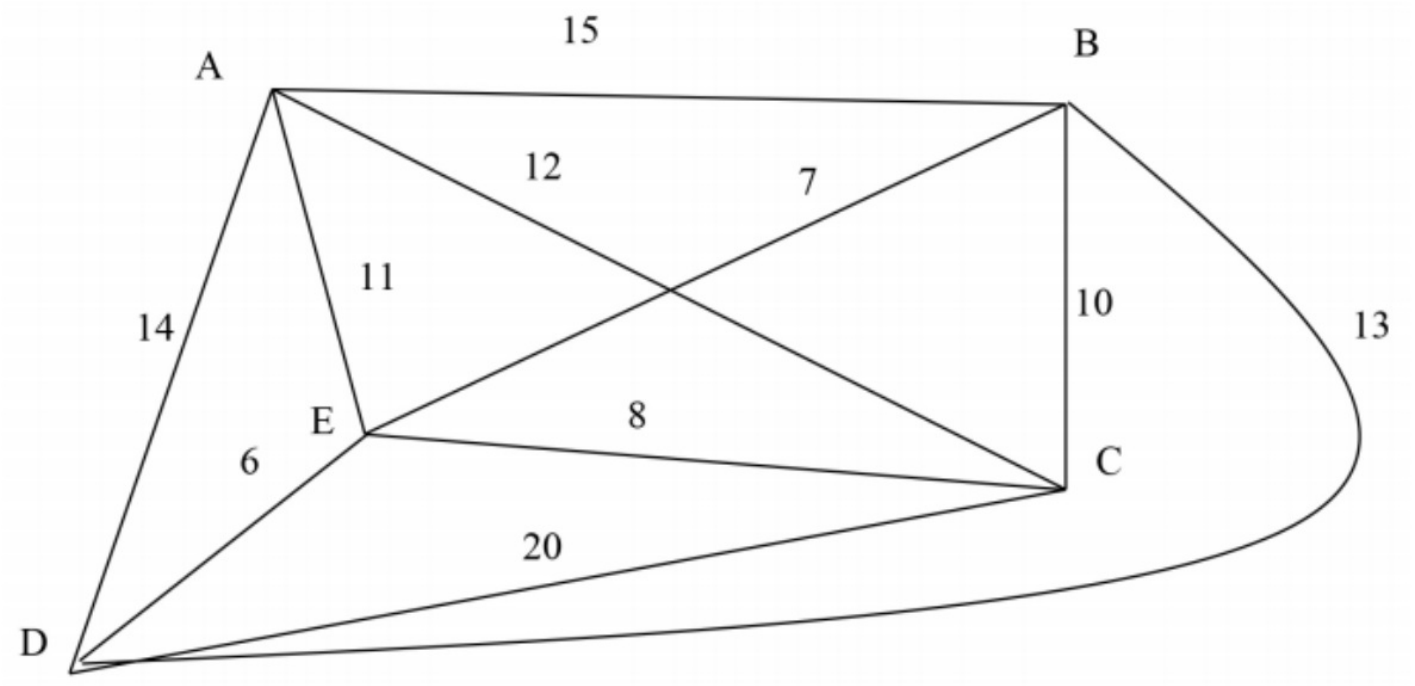
	A	B	C	D	E
A	---	220	150	100	130
B	220	---	160	200	240
C	150	160	---	180	110
D	100	200	180	---	190
E	130	240	110	190	---



Nearest Neighbor Algorithm Practice

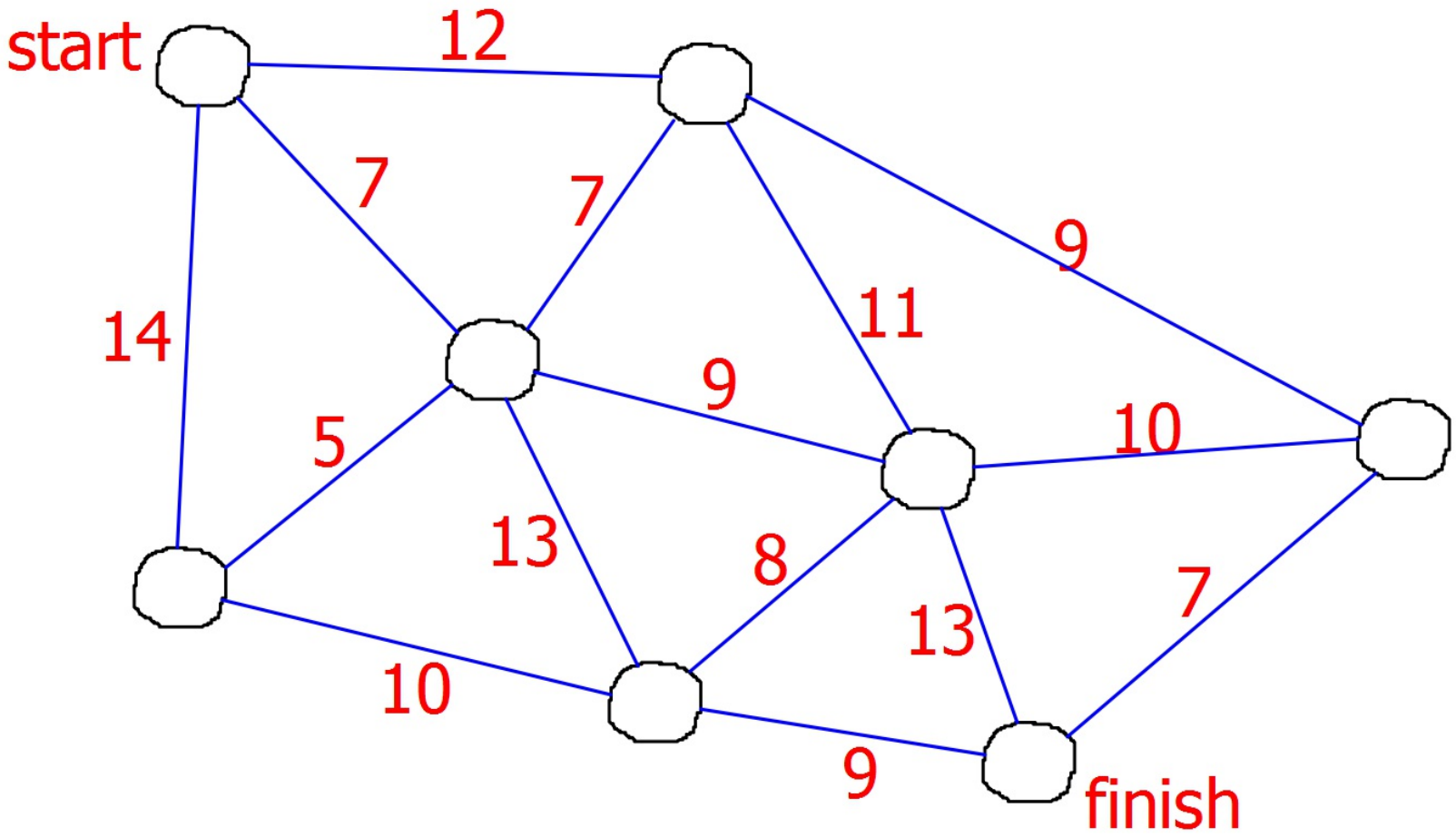
Solve the Traveling Salesman Problem starting at all 5 vertices.

A-
B-
C-
D-
E-



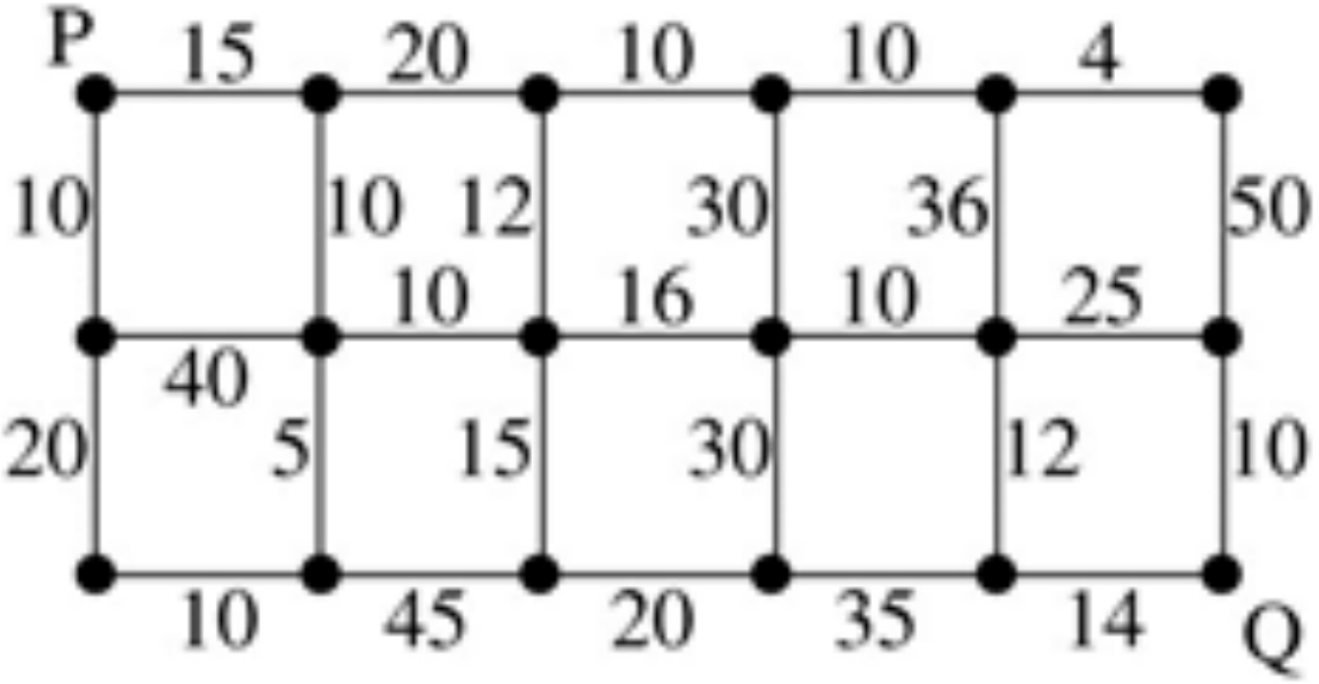
Dijkstra Algorithm

Choice-making method used to find the shortest distance from any 2 points.

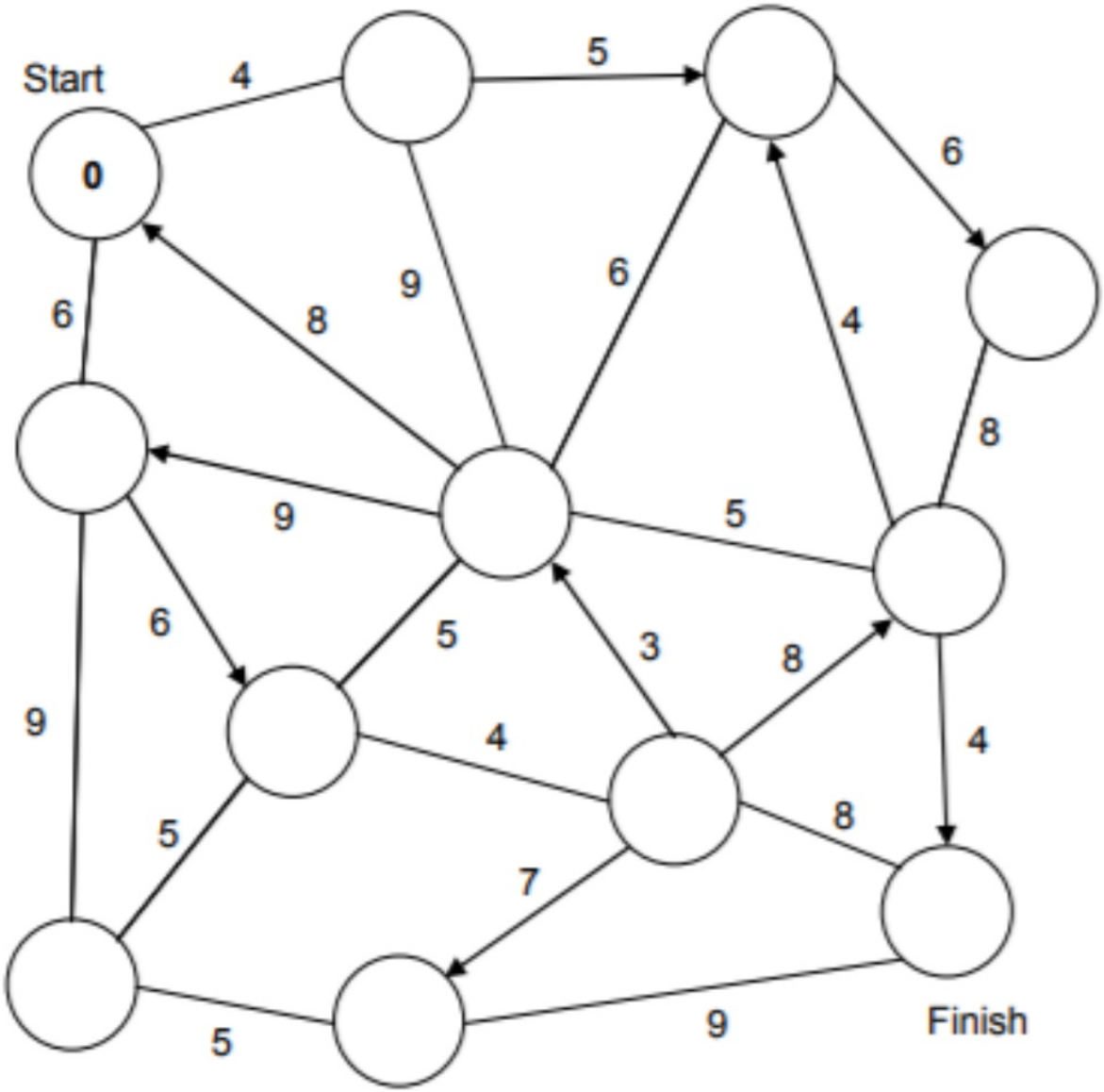


Dijkstra Algorithm Practice

Find the shortest distance from P to Q.



Dijkstra Algorithm Practice



Minimal Spanning Trees Steps

Prim's Algorithm

1) Select a vertex
2) Use the smallest edge to connect new vertex
3) Repeat until all vertices are connected

* if 2 edges are the same weight, choose either

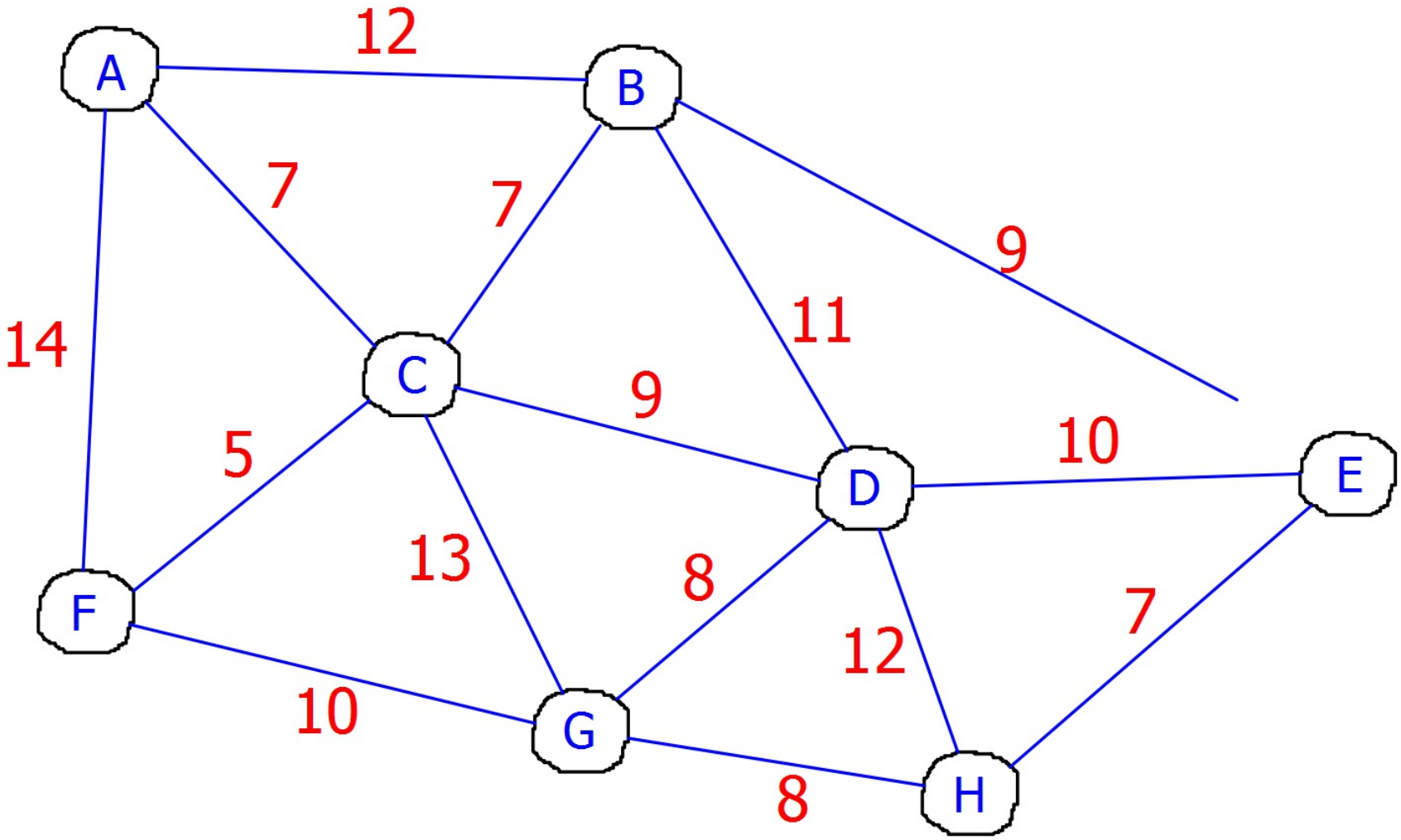
Kruskal's Algorithm

1) Select smallest edge
2) Continue selecting next smallest edge to connect new vertices
3) Repeat until all vertices are connected

* if 2 edges are the same weight, choose either

Minimal Spanning Tree (MST)

Connect all vertices using the minimum edge sum.

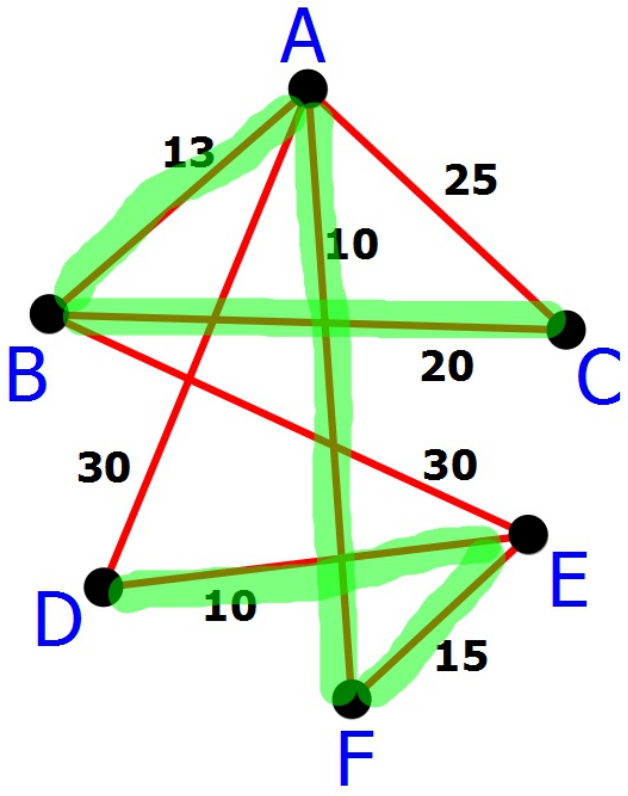


MST Network Problem Example

Draw a network to represent this problem.
Calculate the MST and minimal networking cost.

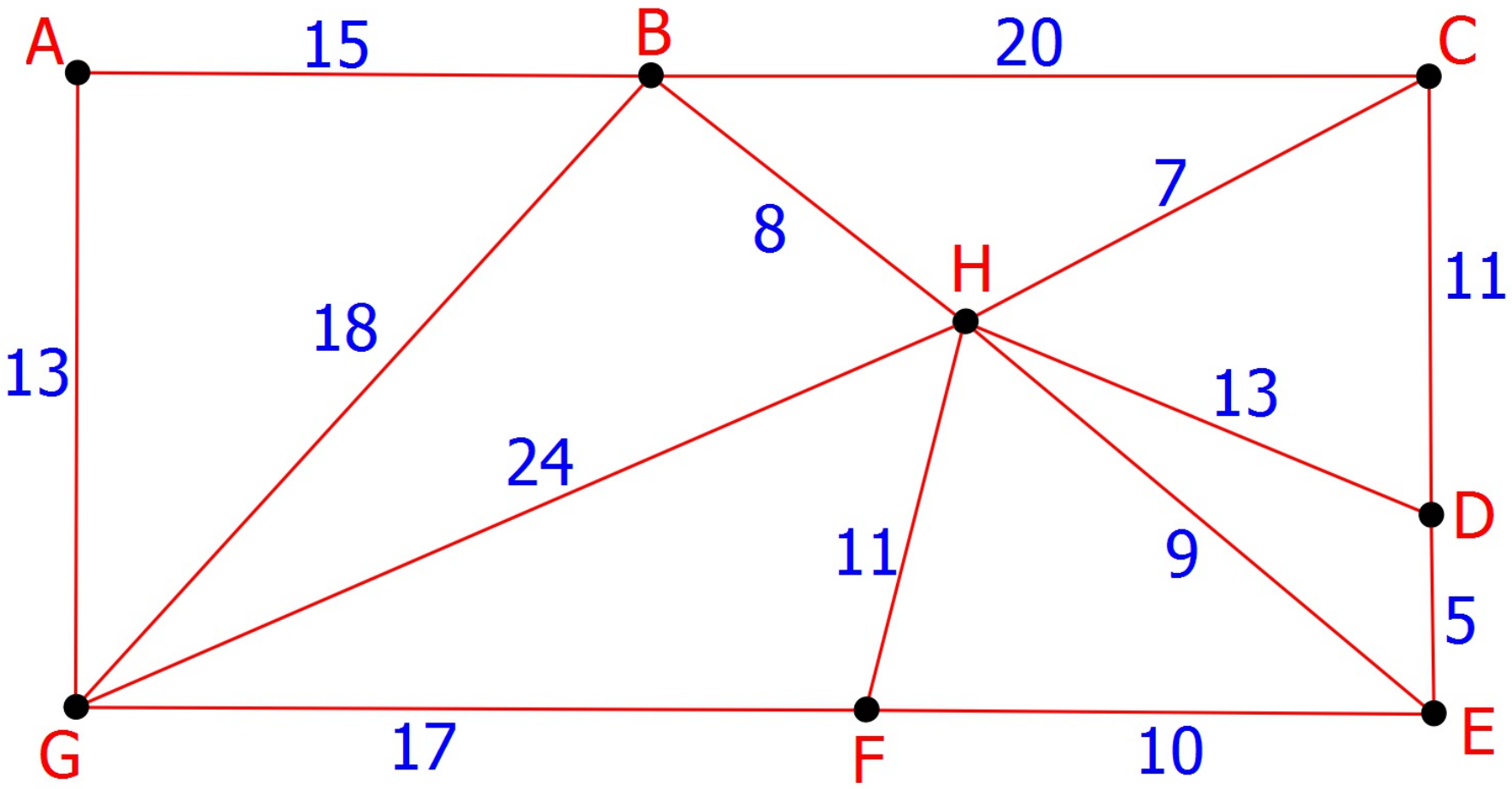
	A	B	C	D	E	F
A		13	25	30		10
B			20		30	
C						
D					10	
E						15
F						

The table above shows the cost, in thousands of dollars, of networking a number of offices at a large complex.



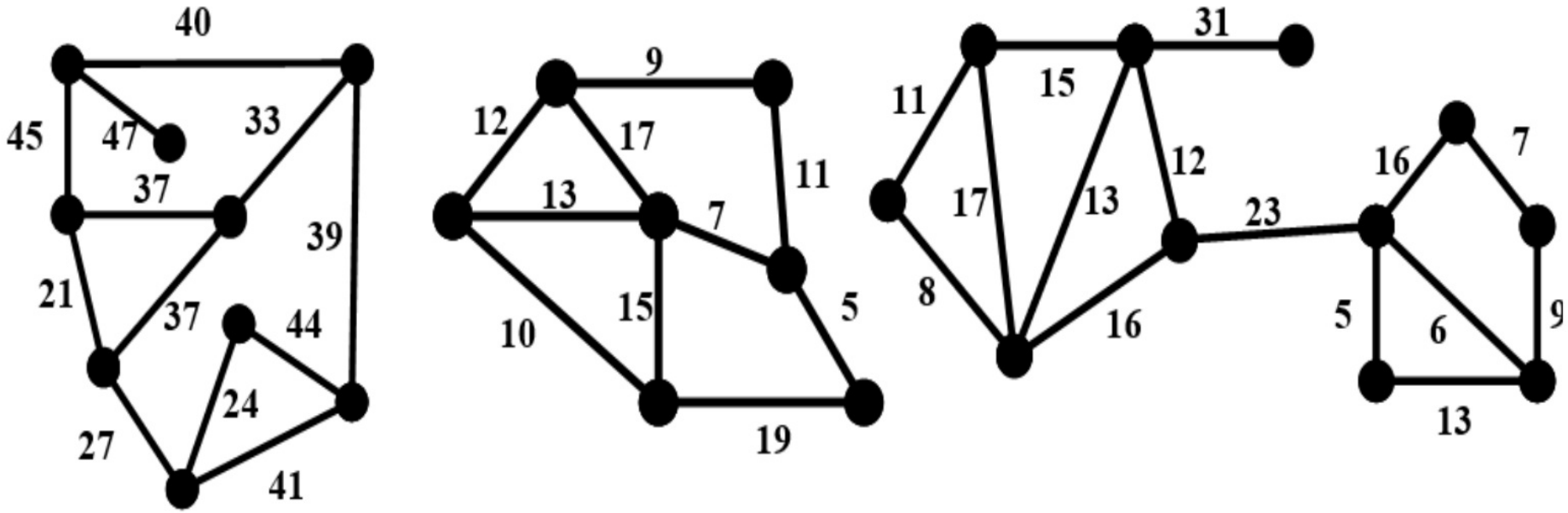
10+10+13+15+20= $68,000

Minimal Spanning Tree (MST) Practice



Minimal Spanning Tree Practice

Calculate the MST of each graph, using either method.



MST Network Problem Practice

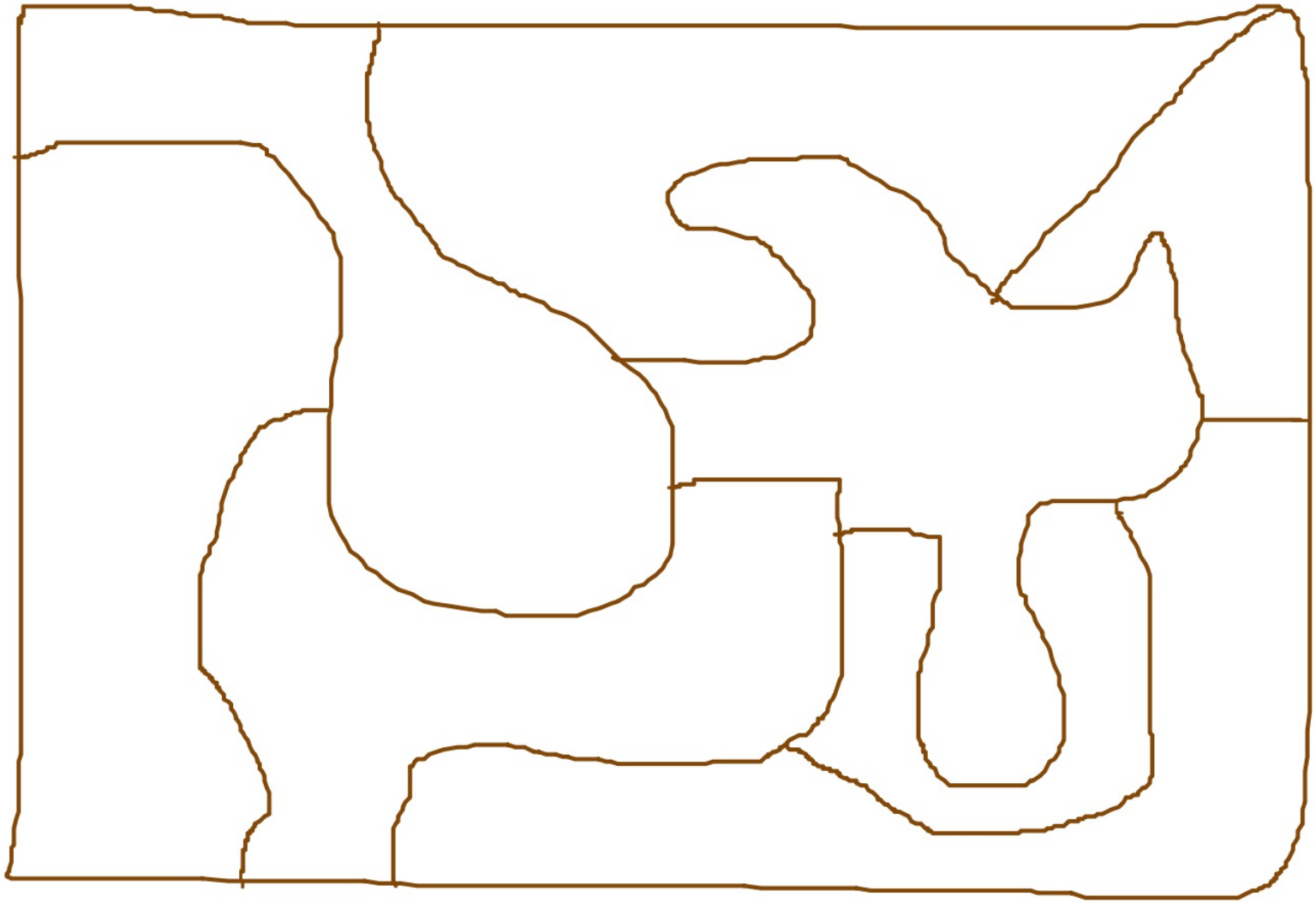
Draw a network to represent this problem.
Calculate the MST and minimal networking miles.

	P	Q	R	S	T	U
P		30	10			
Q			15	5	8	
R				20	5	
S					10	10
T						30
U						

Six towns are connected by roads and the distance between them, in kilometres, is shown in the table.

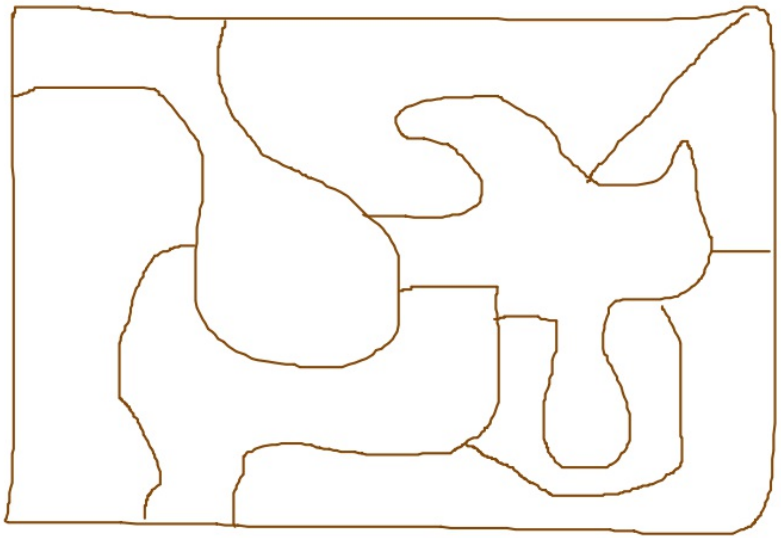
4-Color Mapping

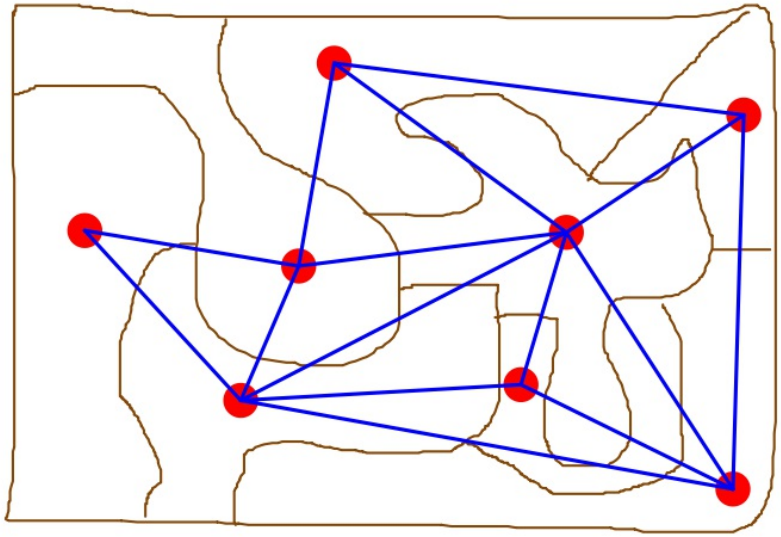
Color every section so that no 2 colors touch



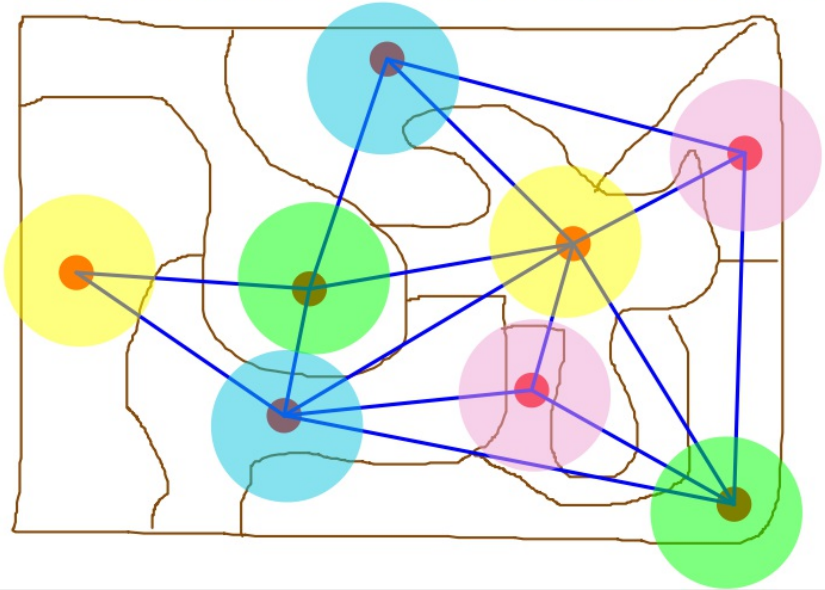
4-Color Mapping Steps

1) Create a graph using borders as edges

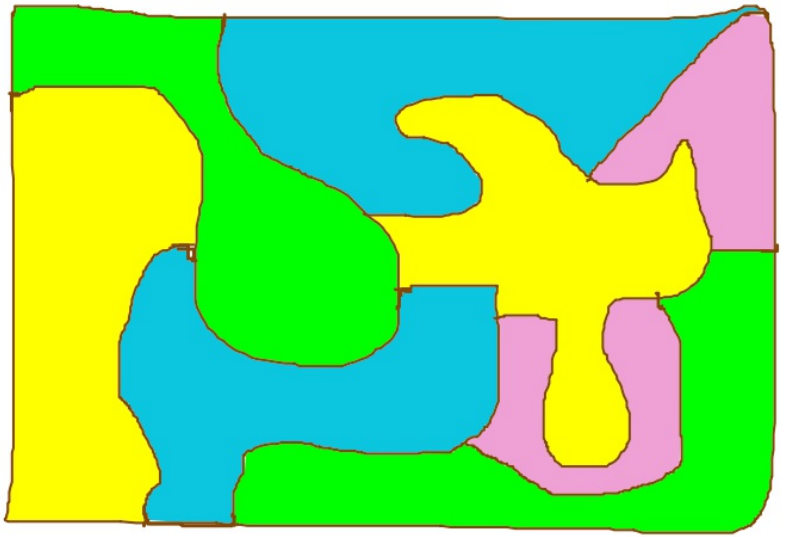




2) Color non-connected the same

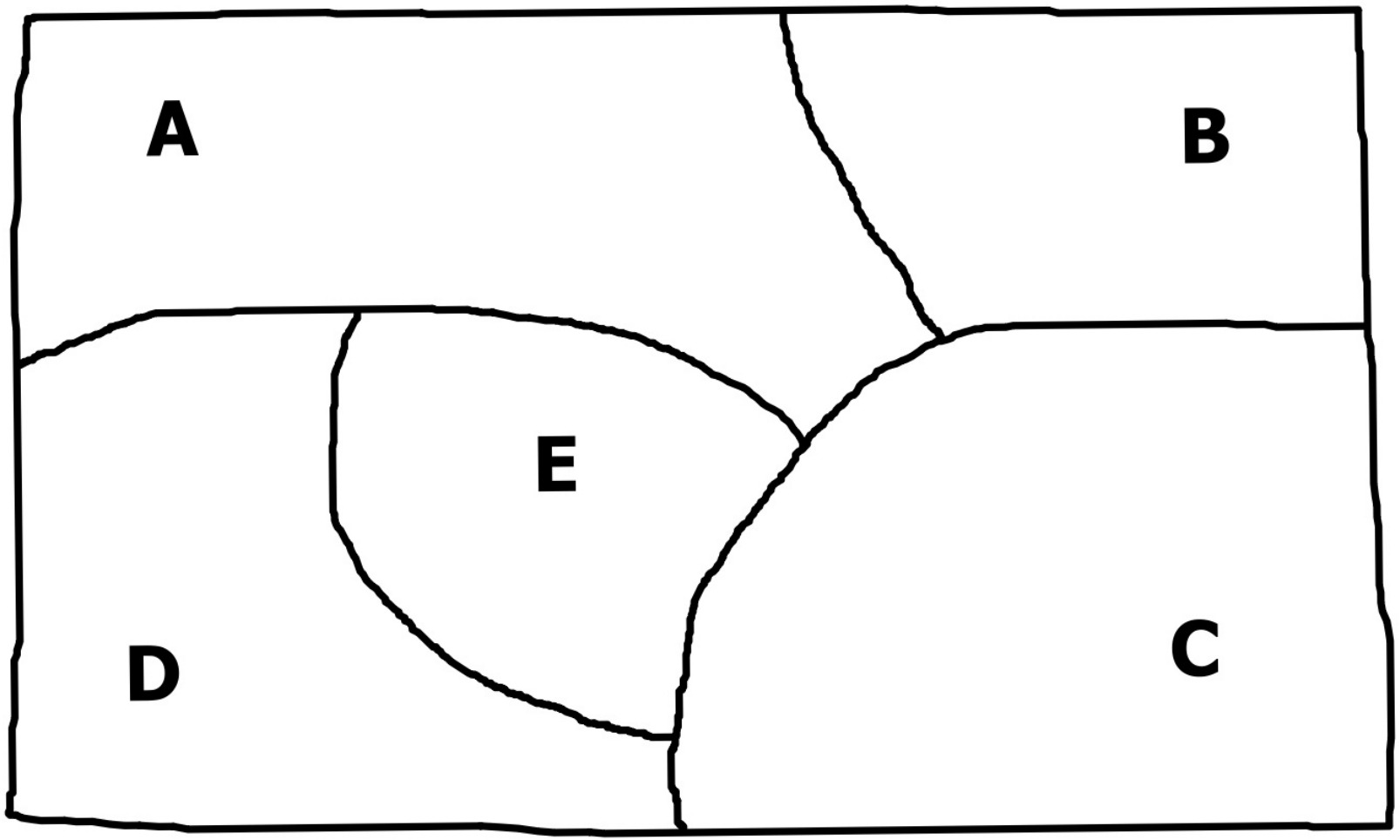


3) Color the graph



4-Color Mapping Practice

Using the minimum number of colors, color every section so that no 2 colors touch

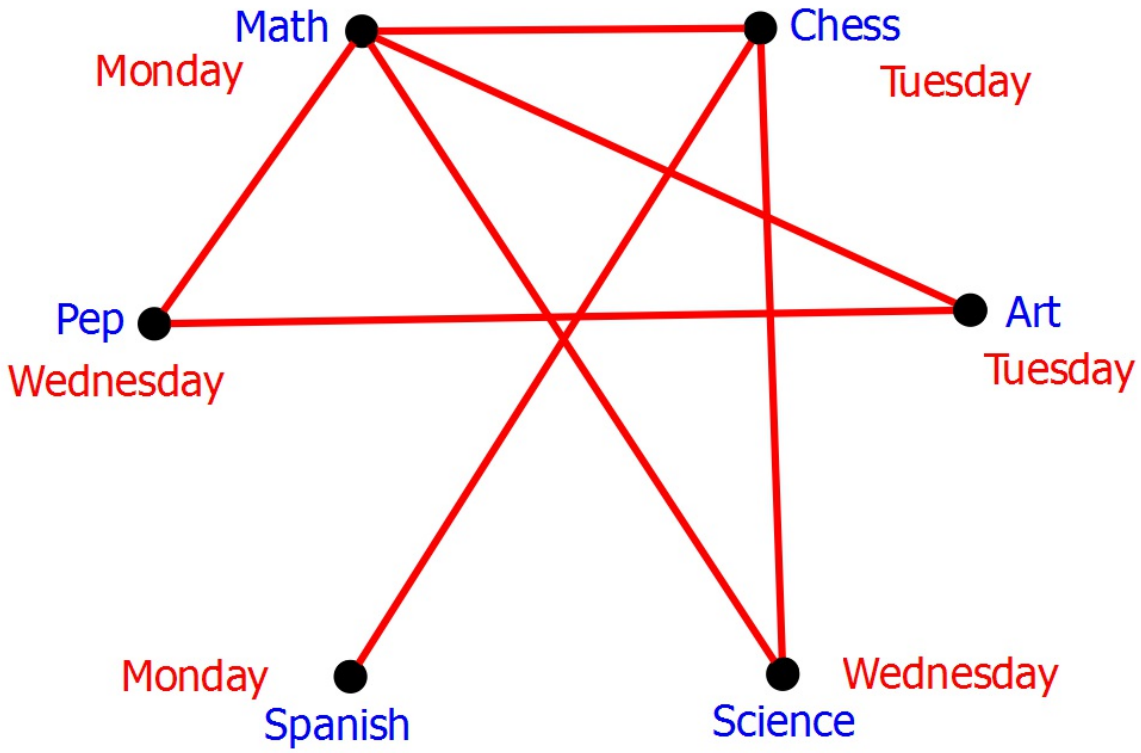


Conflict Maps

1) Create a graph
* vertices represent options
* edges represent conflicts
2) Choose the vertex with the most edges
* label that option as first time period
* label any non-connected vertex as same time
3) Choose unlabeled vertex with next most edges
* label that option as second time period
* label any non-connected vertex as same time
4) Continue until all vertices are labeled

Conflict Map Example

	Math Club	Chess Club	Science Club	Art Club	Pep Club	Spanish Club
Nicole	X	X	X	—	—	—
Mattie	X	—	—	X	X	—
Quin	—	X	—	—	—	X
Emma	X	—	X	—	—	—
Alex	X	—	—	—	X	—



MEETINGS:

Monday:
Math/Spanish

Tuesday:
Chess/Art

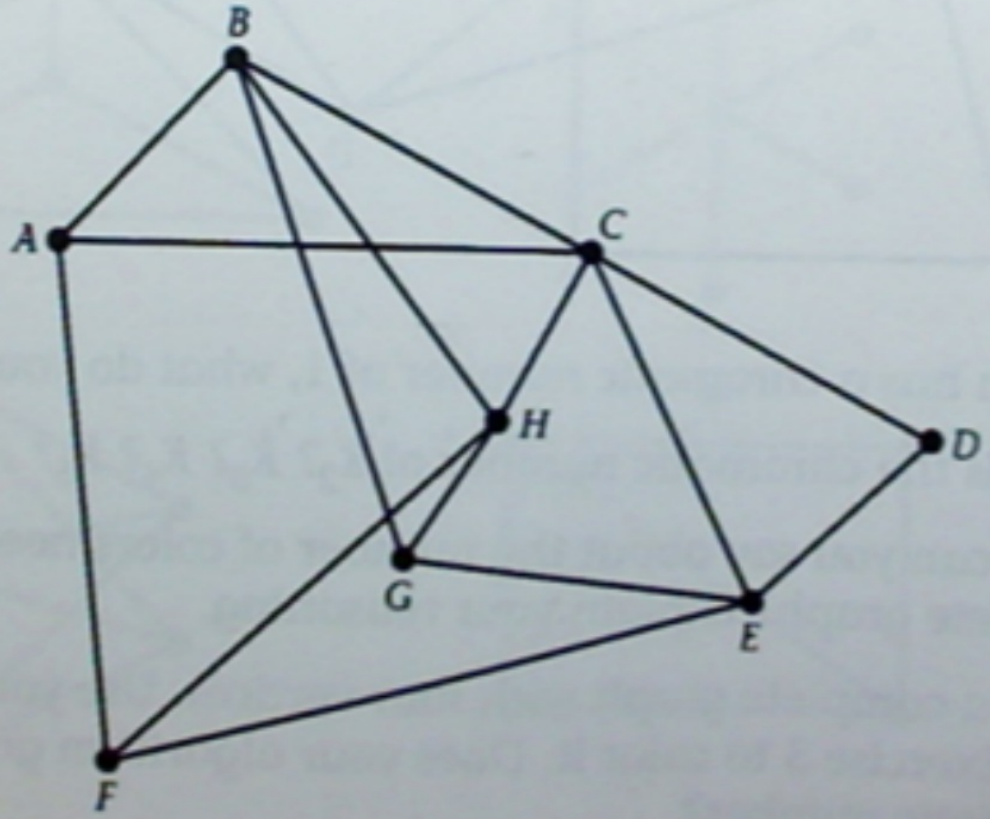
Wednesday:
Science/Pep

Conflict Map Practice

Committee	Adam	Betty	Carl	Don	Eddy	Frank	Gus	Hank	Inez
Calendar	X			X				X	X
Academics	X					X	X	X	
Sports		X				X			X
Music/Art		X	X						X
Neighbors		X		X	X		X	X	
Building			X		X	X	X		

Conflict Map Practice

Ms. Suzuki is planning to take her history class to the art museum. Following is a graph showing those students who are not compatible. Assuming that the seating capacity of the cars is not a problem, what is the minimum number of cars necessary to take the students to the museum?



Critical Paths (Network Diagram)

1) Create task chart

* label and list each task
* determine time for each task
* determine prerequisite tasks

2) Create the digraph

* use relays from start to finish, paying attention to prerequisites and parallel tasks
* use weighted relays for task times

3) Calculate ESTs, trace critical path, calculate MPT

* calculate earliest start times for each task, by adding previous vertex time to relay time (if more than one, always use largest number)
* trace route of ESTs to label critical path

4) Calculate LSTs

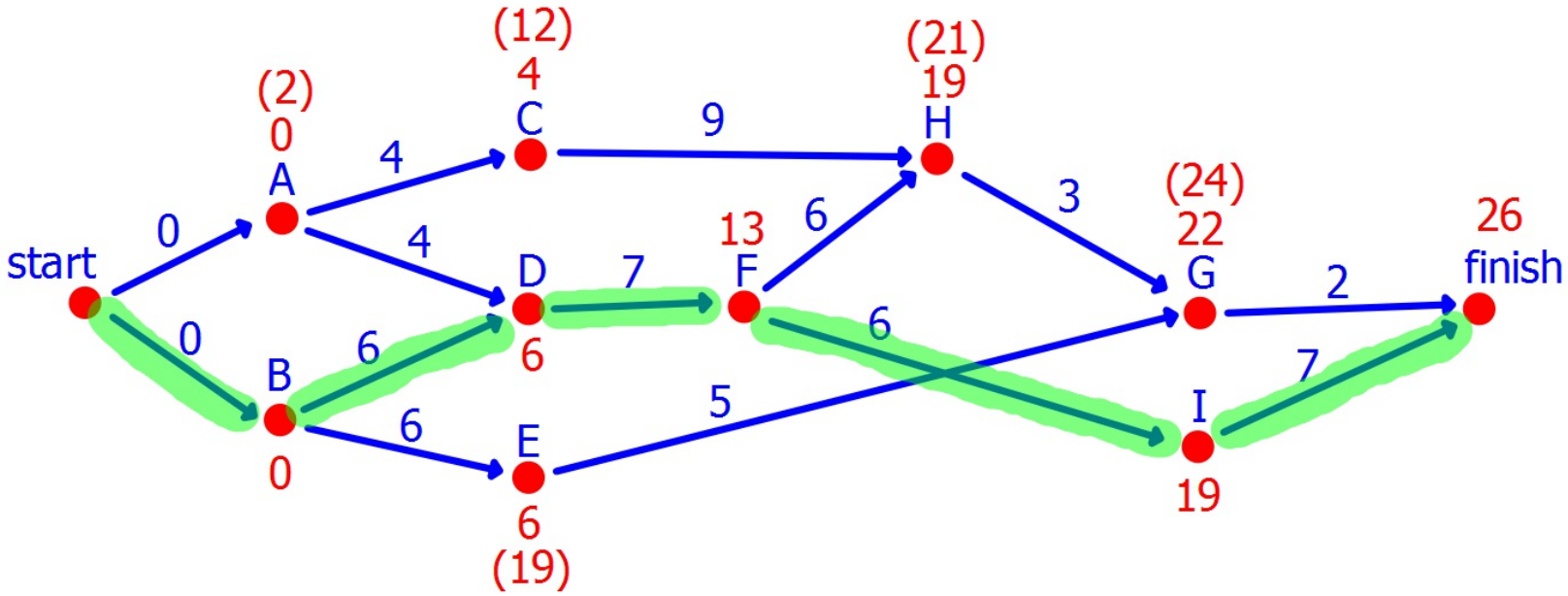
* Trace backwards off critical path to find LSTs

Critical Path Example

Task	Time	Prerequisites
A	4	None
B	6	None
C	9	A
D	7	A, B
E	5	B
F	6	D
G	2	E, H
H	3	C, F
I	7	F
Finish		G, I

Critical Path:
B, D, F, I

Minimal
Project Time:
26



Critical Path Practice

Task	Time	Prerequisites
Start	0	—
A	5	None
B	8	*A, D*
C	9	*B, I*
D	7	None
E	8	*B*
F	12	*I*
G	4	*C, E, F*
H	9	None
I	5	*D, H*
Finish		

Critical Path:

Minimal Project Time:

Critical Path Practice

	Task	Time (days)	Prerequisite Task
Start		0	—
A	Buy film	1	None
B	Load camera	1	*A*
C	Take photos of clubs	3	*B*
D	Take sports photos	2	*C*
E	Take photos of teachers	1	*B*
F	Develop film	2	*D, E*
G	Design layout	5	*D, E*
H	Print and mail pages	3	*G, F*

Critical Path:

Minimal Project Time:

Critical Path Practice

Task label	Task	Time (minutes)	Prerequisite
Start	Decide to begin	0	None
A	Slice ham	7	None
B	Heat ham in microwave	10	A
C	Peel and cut yams	8	None
D	Melt butter and brown sugar	5	None
E	Cook yams	20	C, D
F	Rinse and trim green beans	7	None
G	Heat water for green beans	6	None
H	Cook green beans	6	F, G
I	Peel and cut potatoes	8	None
J	Heat water for potatoes	10	None
K	Cook potatoes	15	I, J
L	Mash and season potatoes	5	K
M	Set table	5	None
N	Plate and serve food	3	B, E, H, L, M
Eat	End of preparation	0	N

Critical Path:

Minimal Project Time: