

# Voting Power

**Weighted Voting:** a situation where voters (players) have an unequal number of votes

**Quota:** the number of votes needed to win

**Dictator:** the player who can reach quota without help

**Dummy:** players who have no voting power

**Coalition:** players that vote together

**Winning Coalition:** a coalition that reaches quota

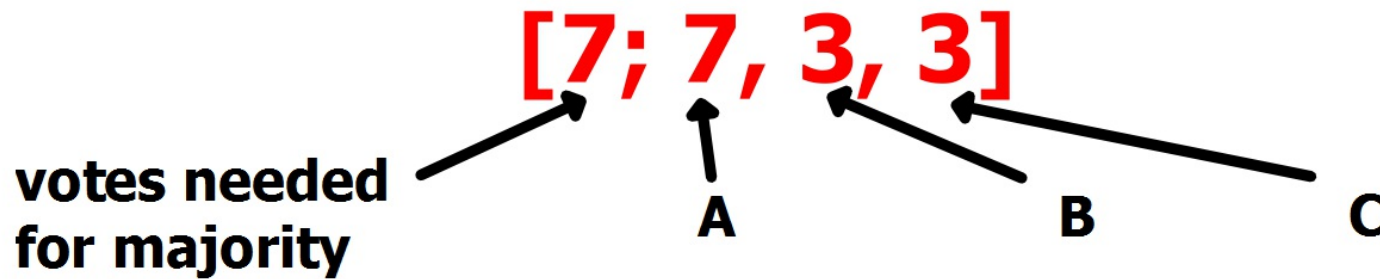
**Veto Power:** occurs when a player who is included in every winning coalition

**Banzhaf Power Index:** a measure of the power of the voter using percentages of critical voters in all winning coalitions

**Shapley-Shubik Power Index:** a measure of the power of the voter by calculating the percentage of the pivotal players in each permutation of the grand coalition

# Weighted Voting

In a situation with three voters, A, B, and C, they have 7, 3, and 3 votes respectively. If a majority is required to pass, is there a dictator? Are there any dummies?



To find the Banzhaf Power Index, write out all winning coalitions, underline all critical voters, then divide the number of times a voter is critical by the total number of critical votes.

Winning coalitions:  $(\underline{7})$ ,  $(\underline{7}, 3_1)$ ,  $(\underline{7}, 3_2)$ , and  $(\underline{7}, 3_1, 3_2)$

$7$ :  $4/4 = 100\%$  Dictator  
 $3_1$ :  $0/4 = 0\%$  Dummy  
 $3_2$ :  $0/4 = 0\%$  Dummy

# Power Indexes

A small high school has 110 students: 50 sophomores, 30 juniors, and 30 seniors. A representative was chosen from each class and given 5, 3, and 3 votes respectively. Which class has the most voting power? Is there a dictator? Are there any dummies?



To find the Banzhaf Power Index, write out all winning coalitions and underline all the critical (essential) voters in each coalition. Then divide the number of times each voter is critical by the TOTAL number of critical votes.

**Winning coalitions:** (5, 3<sub>1</sub>), (5, 3<sub>2</sub>), (3<sub>1</sub>, 3<sub>2</sub>), and (5, 3<sub>1</sub>, 3<sub>2</sub>)

$$5: 2/6 = 33.3\%$$

$$3_1: 2/6 = 33.3\%$$

$$3_2: 2/6 = 33.3\%$$

# Power Indexes

To find the Shapley-Shubik Power Index (different method), write out each permutation of all the players. Then starting on the left, add players' votes until you reach quota. The player that reaches quota is considered PIVOTAL. Find the power by dividing the number of times each player is pivotal by the number of times ANY player is pivotal.

**[6; 5, 3, 3]**

**53<sub>1</sub>3<sub>2</sub>**

**53<sub>2</sub>3<sub>1</sub>**

**3<sub>1</sub>53<sub>2</sub>**

**3<sub>1</sub>3<sub>2</sub>5**

**3<sub>2</sub>53<sub>1</sub>**

**3<sub>2</sub>3<sub>1</sub>5**

$$5: 2/6 = 33.3\%$$

$$3_1: 2/6 = 33.3\%$$

$$3_2: 2/6 = 33.3\%$$

# Power Indexes

## A Power Index Algorithm

1. List all coalitions of voters that are winning coalitions.
2. Select any voter, and record a 0 for that voter's power index.
3. From the list in step 1, select a coalition of which the voter selected in step 2 is a member. Subtract the number of votes the voter has from the coalition's total. If the result is less than the number of votes required to pass an issue, add 1 to the voter's power index.
4. Repeat step 3 until all coalitions of which the voter chosen in step 2 is a member are checked.
5. Repeat steps 2 through 4 until all voters are checked.